

Integrating large- to high- dimension markers in mechanistic models



Mélanie Prague, Auriane Gabaut, Lisa Crépin, Boris Hejblum, Rodolphe Thiébaut and Cécile proust-Lima

Pharmacometrics in France – 19 Sept. 2025



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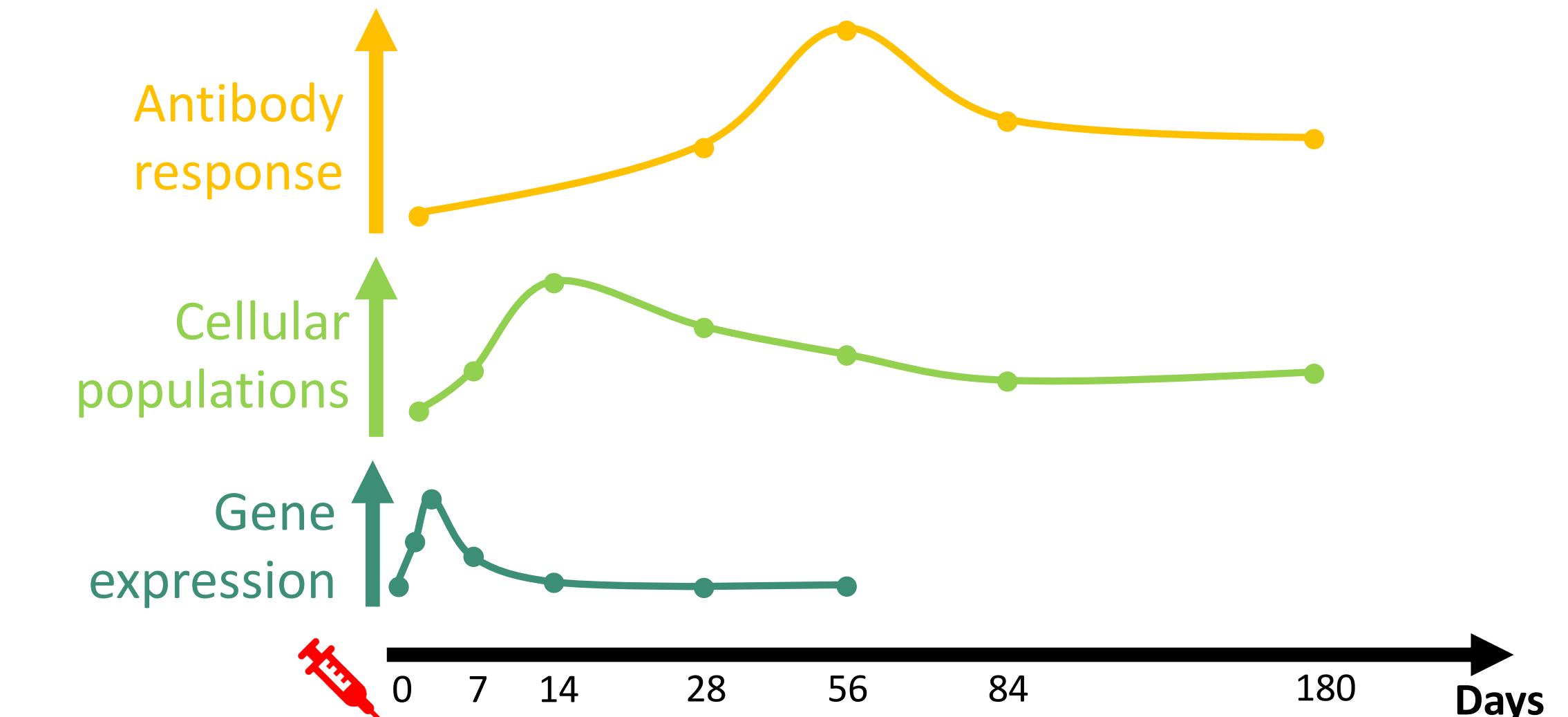
01

INTRODUCTION AND MOTIVATION

Motivations - Vaccinometrics

Starting point : Vaccine development for treatment and control of infectious diseases

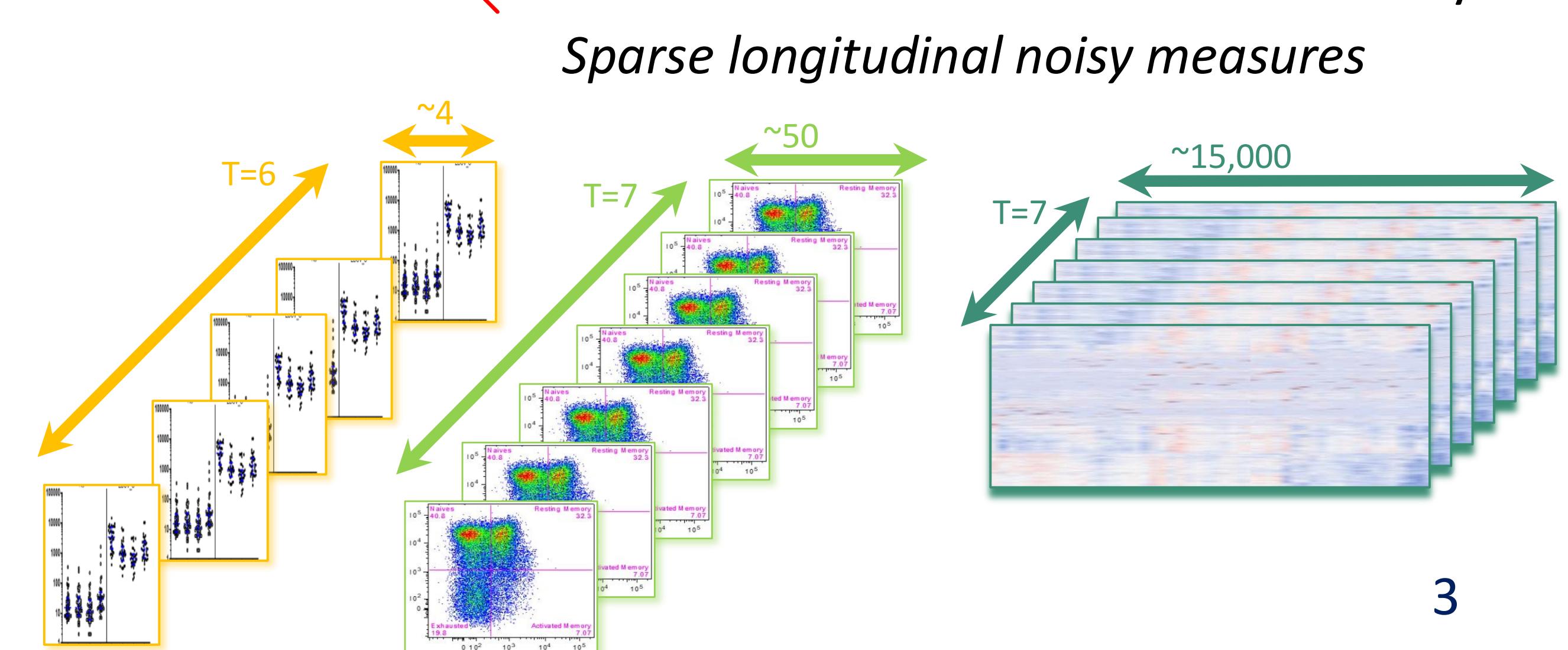
How it works ?



How long ?

How to predict the response
in each individual ?

What is the optimal
vaccination strategy ?



Mechanistic models – Population approach

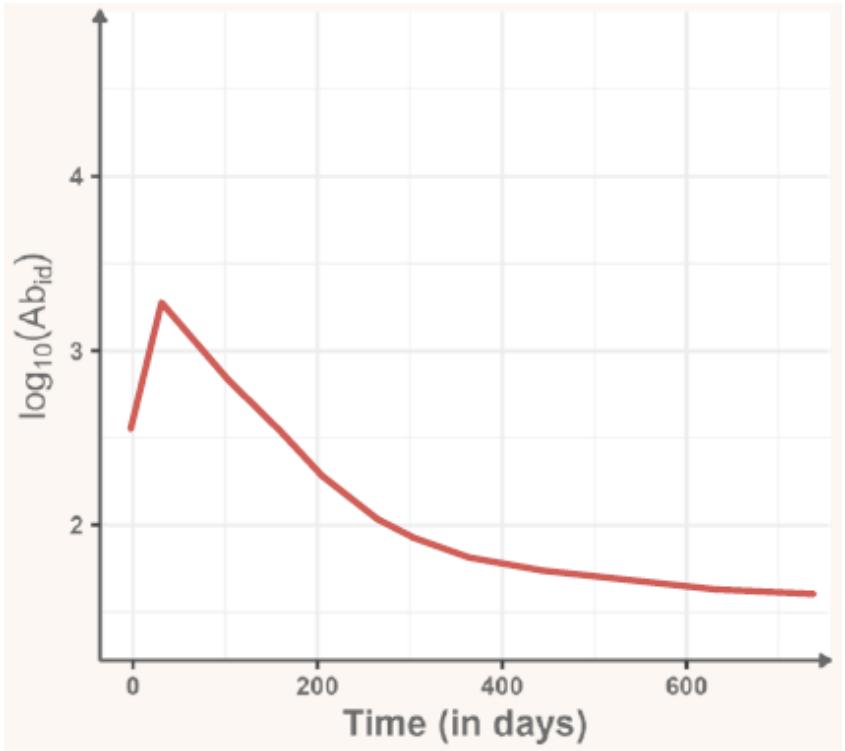
Structural model (ODE-based)

$$\begin{cases} \frac{dX_k}{dt} = f_k(X_1, \dots, X_K, \theta, t), & X_k \in X \\ X_k(t=0) = X_{k,0}, & k \in \{1, \dots, K\} \end{cases}$$

Mechanistic models – Population approach

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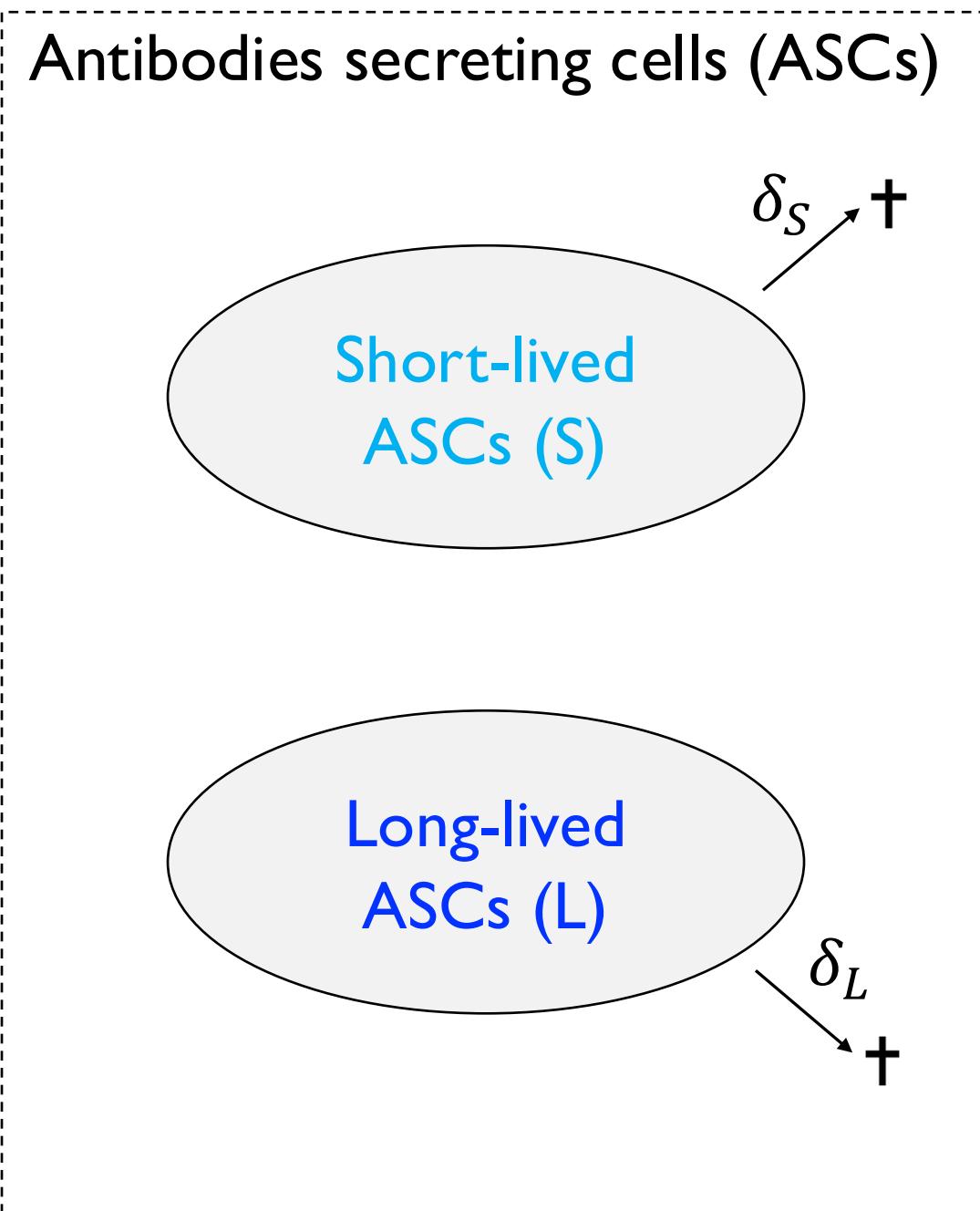
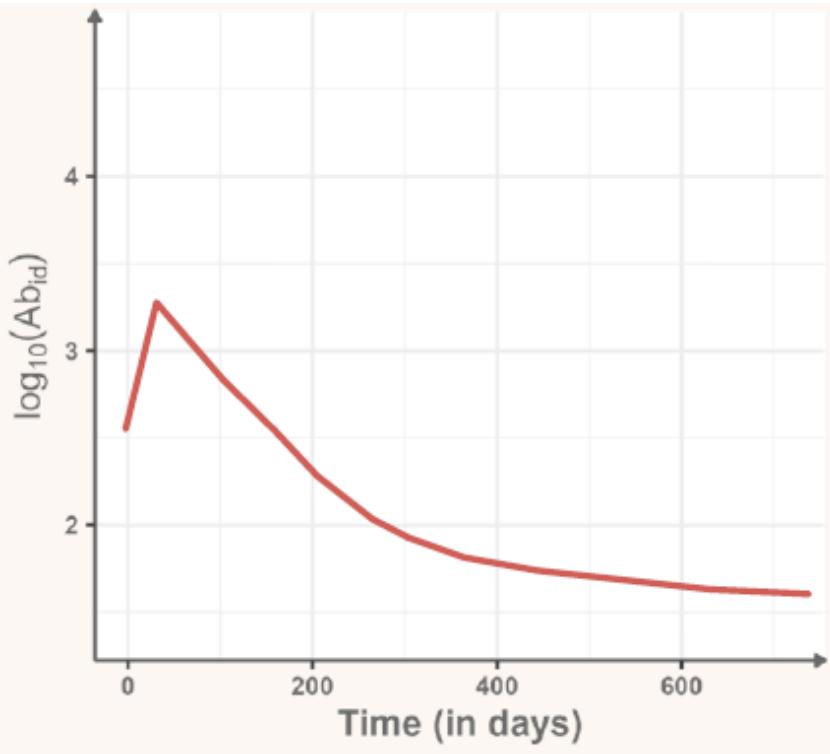
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Mechanistic models – Population approach

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$$\frac{dS}{dt} = -\delta_S S$$

$$\frac{dL}{dt} = -\delta_L L$$

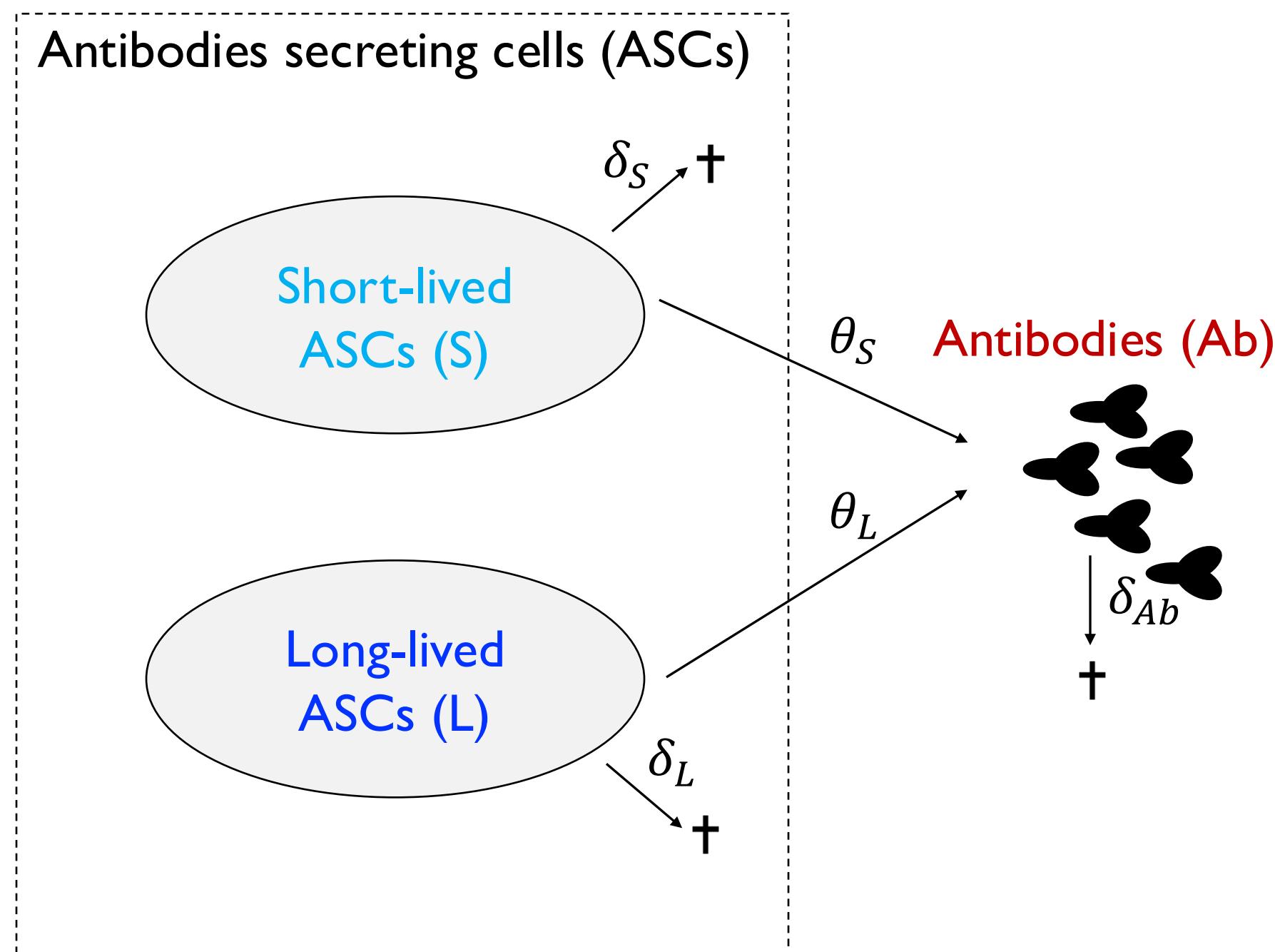
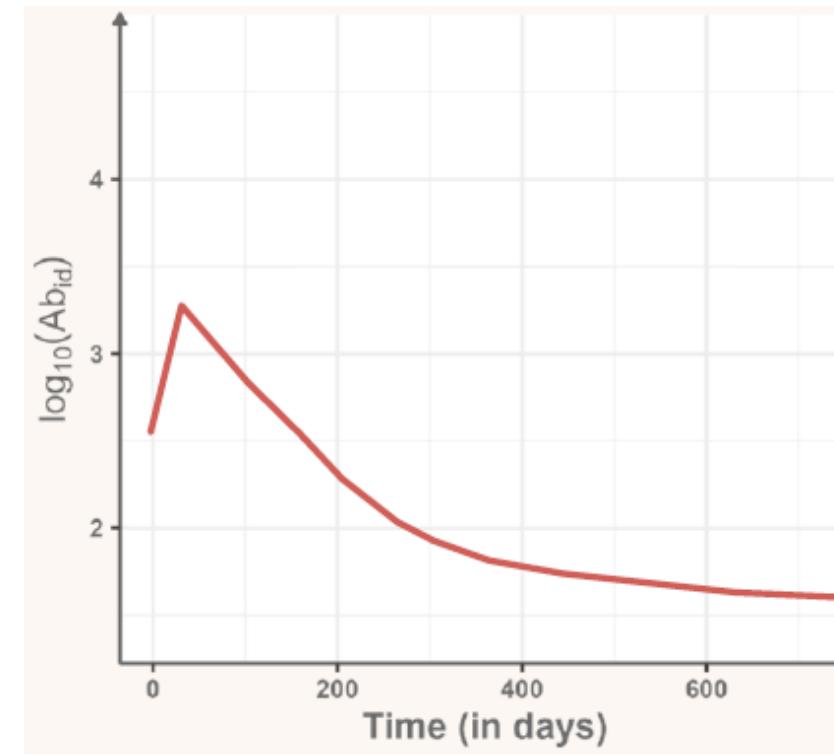
$$S(t=0) = S_0$$

$$L(t=0) = L_0$$

Mechanistic models – Population approach

□ Structural model (ODE-based)

$$\begin{cases} \frac{dX_k}{dt} = f_k(X_1, \dots, X_K, \theta, t), & X_k \in X \\ X_k(t=0) = X_{k,0}, & k \in \{1, \dots, K\} \end{cases}$$



$$\frac{dS}{dt} = -\delta_S S$$

$$S(t=0) = S_0$$

$$\frac{dL}{dt} = -\delta_L L$$

$$L(t=0) = L_0$$

$$\frac{dAb}{dt} = \theta_S S + \theta_L L - \delta_{Ab} Ab$$

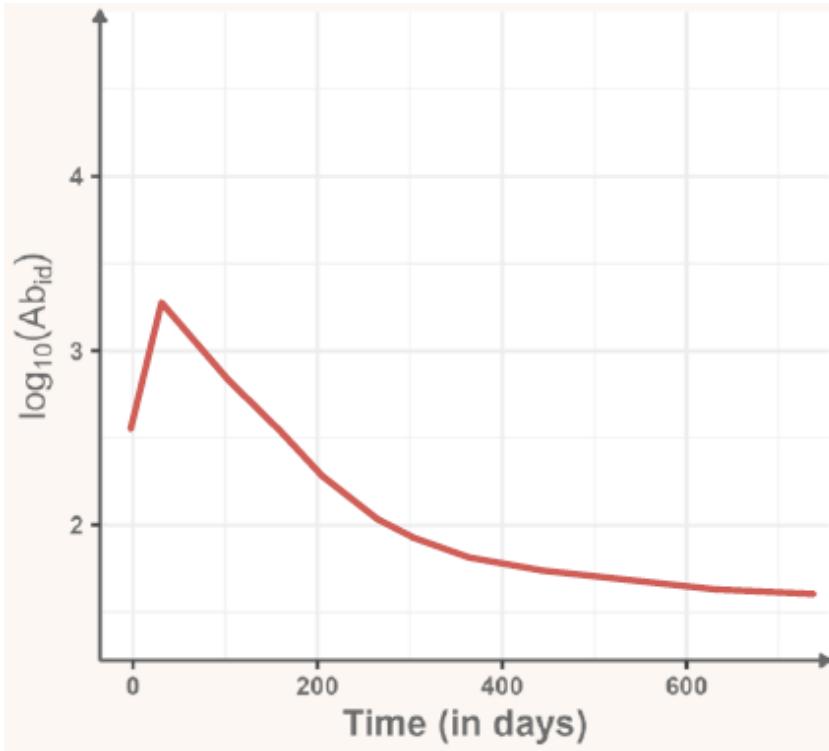
$$Ab(t=0) = 0$$

$$\theta = (\delta_S, \delta_L, \theta_S, \theta_L, \delta_{Ab})$$

Mechanistic models – Population approach

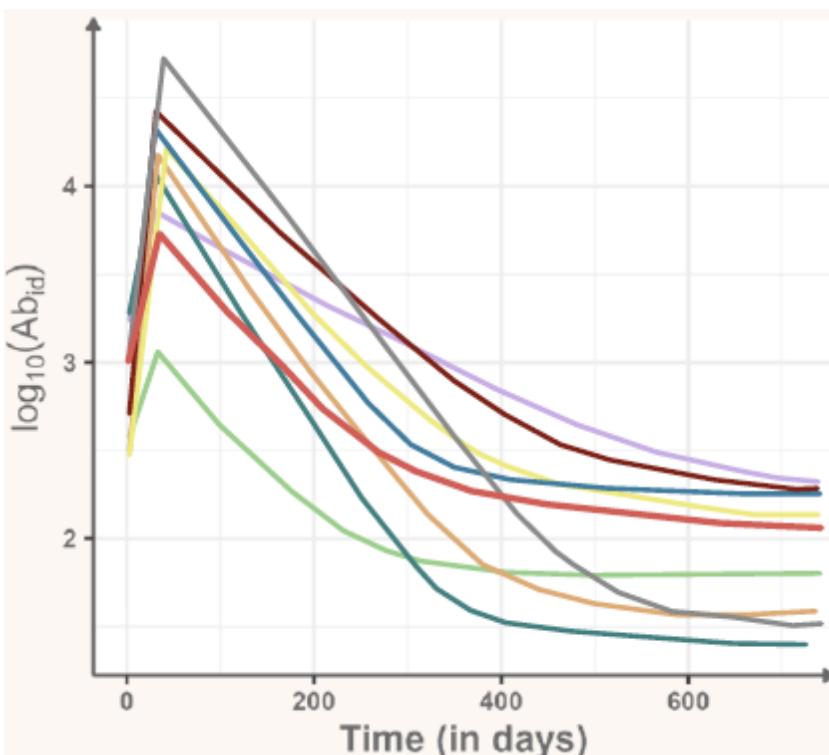
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□ Statistical model (linear mixed effect model)

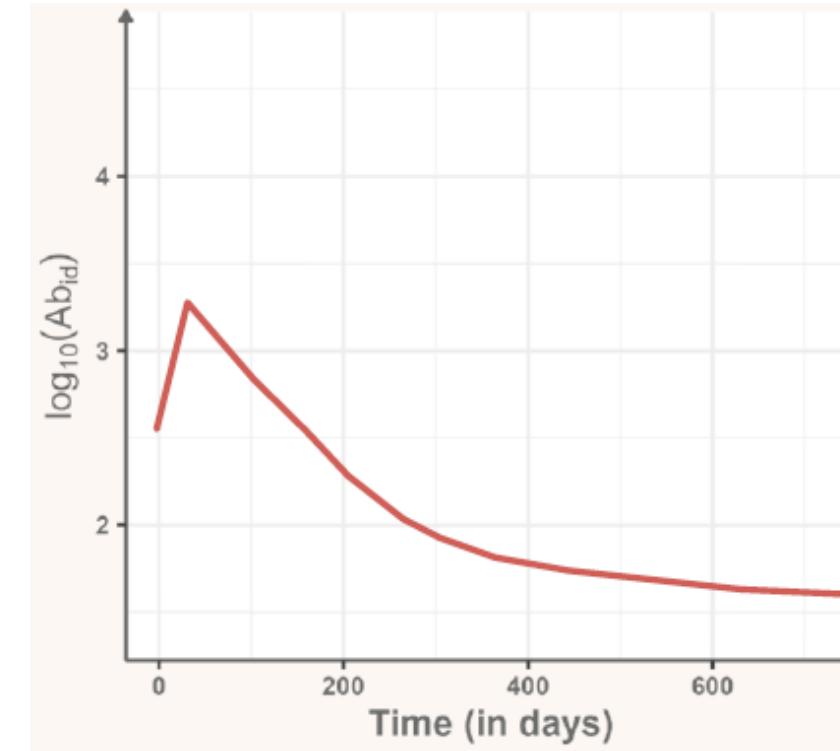
$$\begin{aligned} g(\theta_i(t)) &= g(\theta_0) + \phi Z_i(t) + u_i \\ u_i &\sim N(0, \Omega), \quad i \in \{1, \dots, N\} \end{aligned}$$



Mechanistic models – Population approach

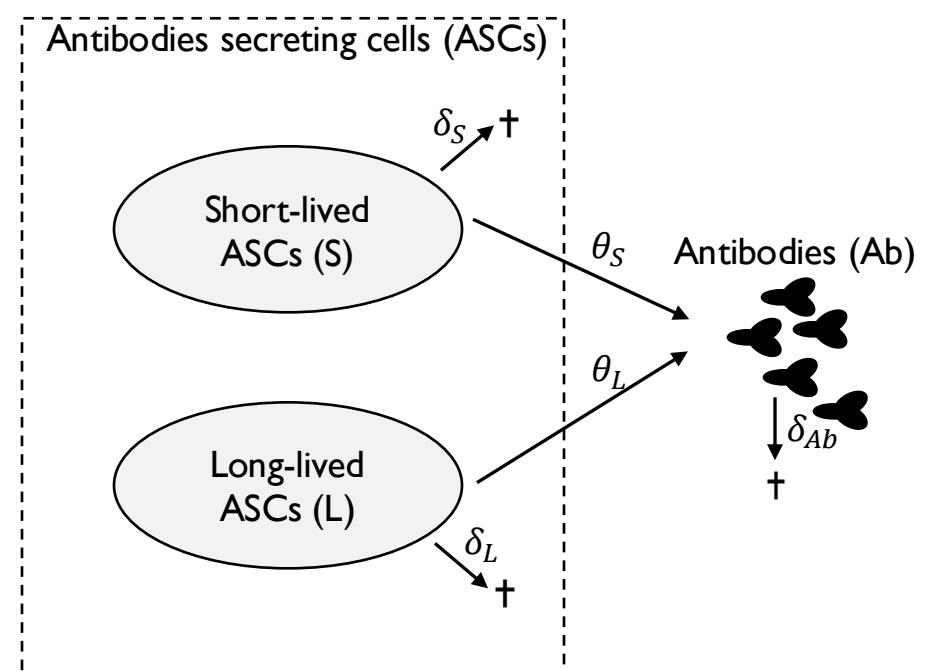
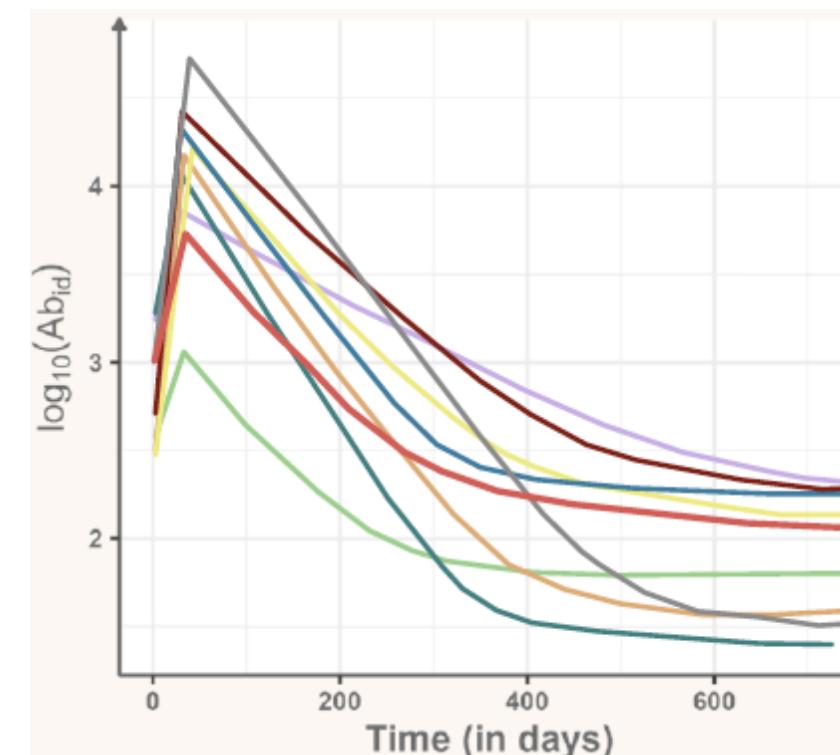
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Ensure positiveness of rates

$$\log(\theta_{S,i}) = \log(\theta_{S,0}) + \beta_{AGE} AGE_i + u_i$$

Population average value

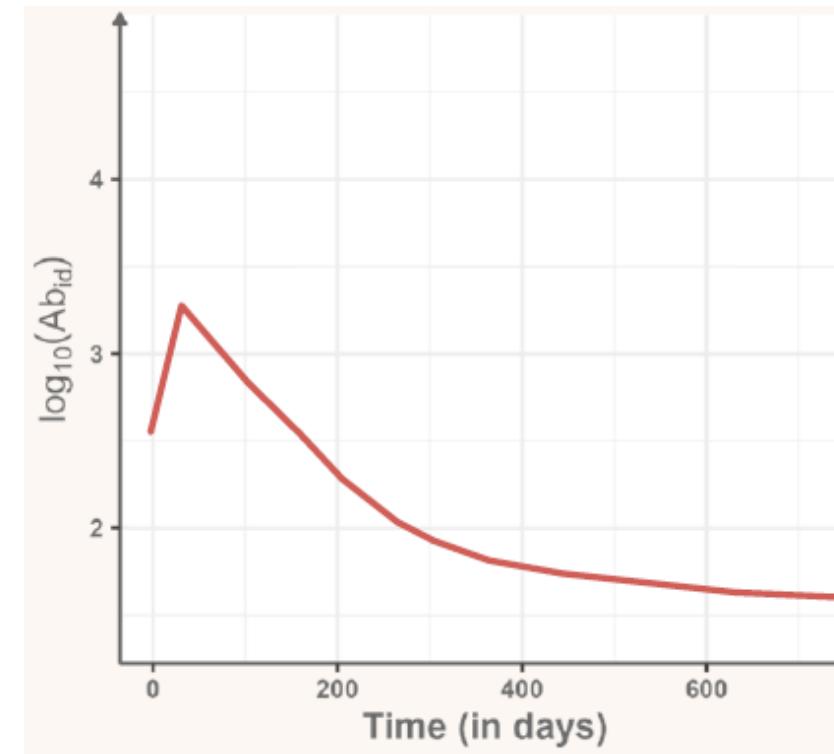
Effect of covariates

Residual heterogeneity
Random effect

Mechanistic models – Population approach

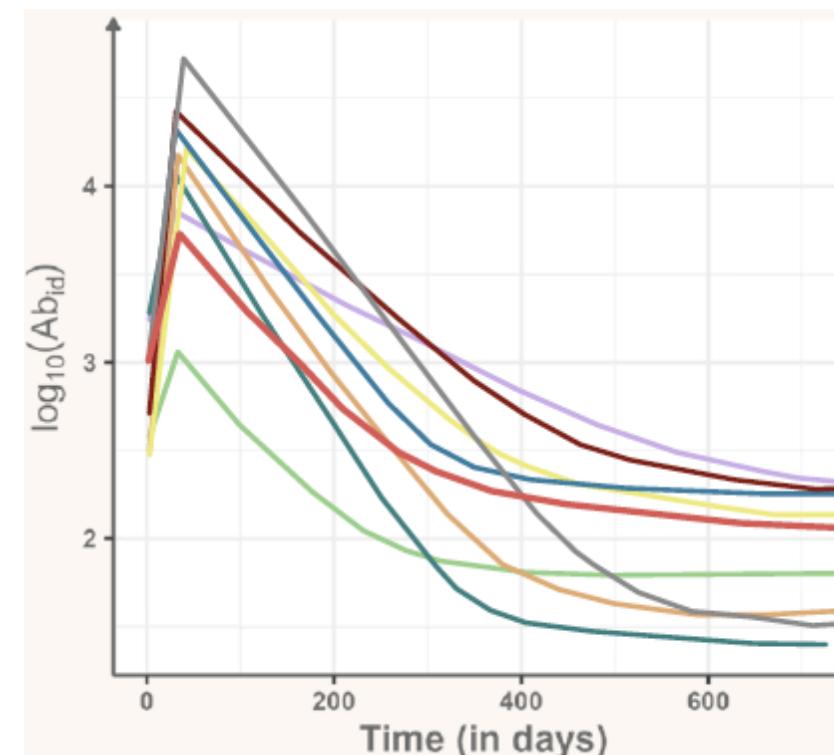
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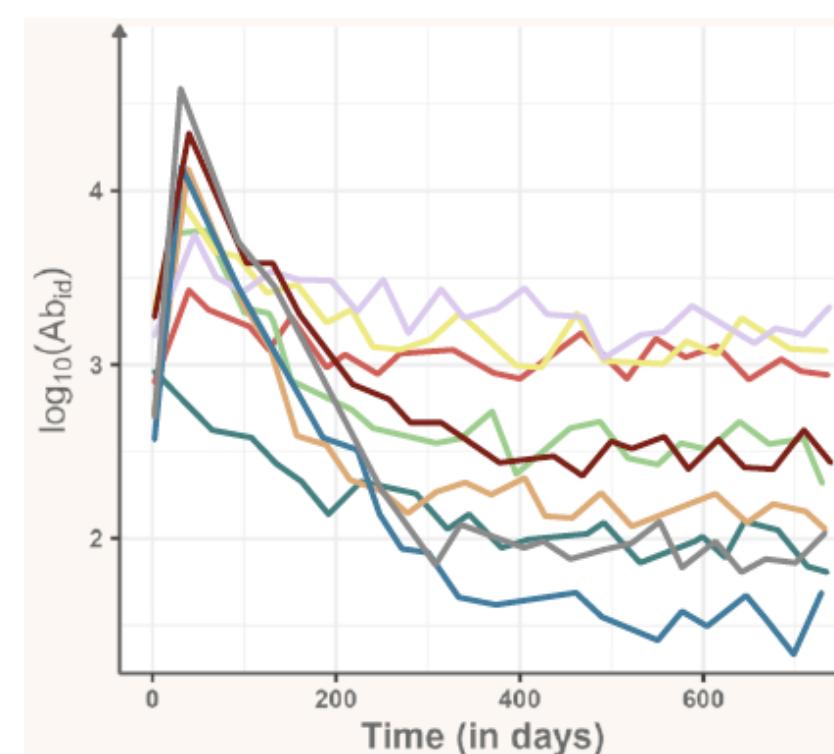
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□ **Observation model** (Error model)

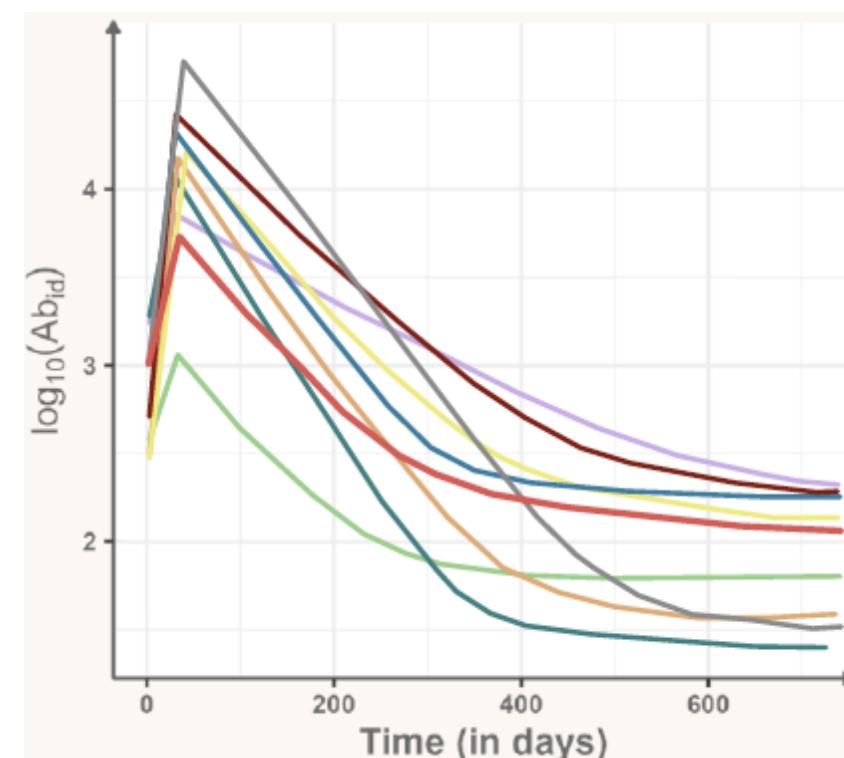
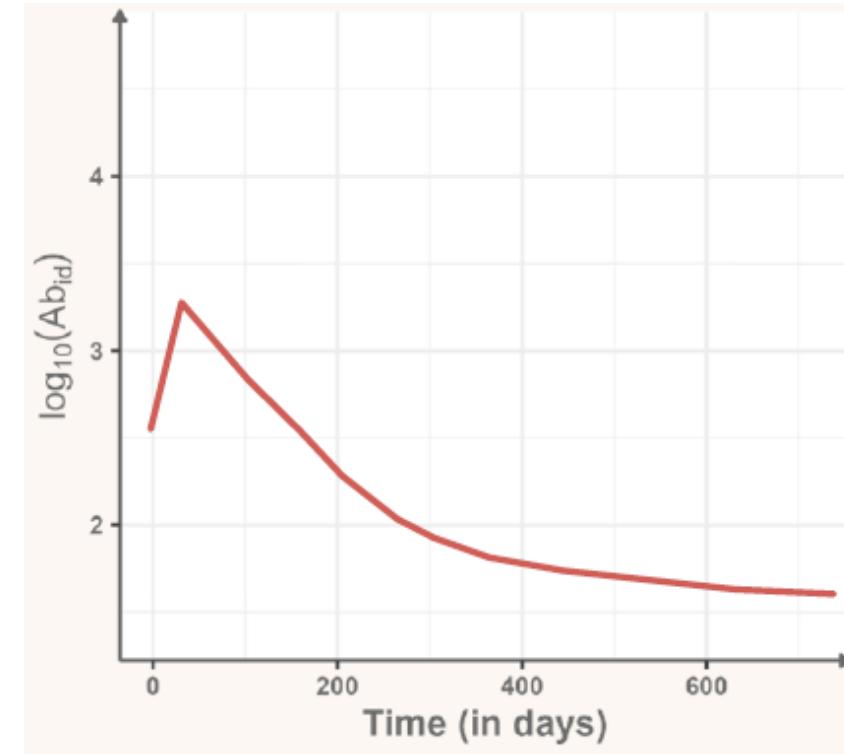
$$\begin{aligned} Y_{ij} &= Y(t_{ij}) = h(X(t_{ij}, \theta)) + g(X(t_{ij}, \theta), \Sigma) \epsilon_{ij} \\ \epsilon_{ij} &\sim N(0, 1) \end{aligned}$$



Mechanistic models – Population approach

□ **Structural model (ODE-based)**

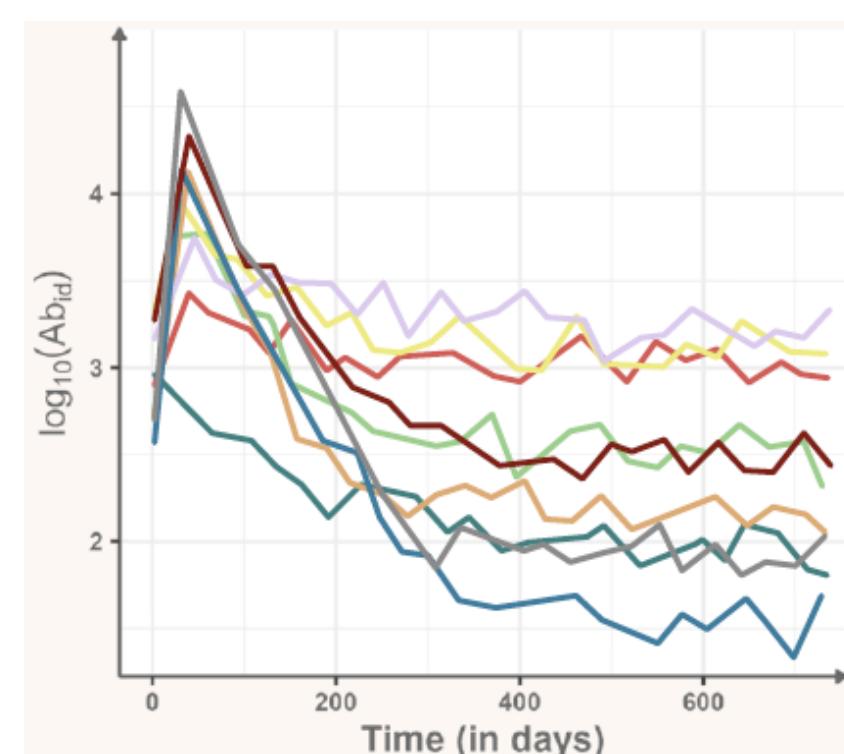
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□ **Statistical model (linear mixed effect model)**

$$g(\theta_i(t)) = g(\theta_0) + \phi Z_i(t) + u_i$$

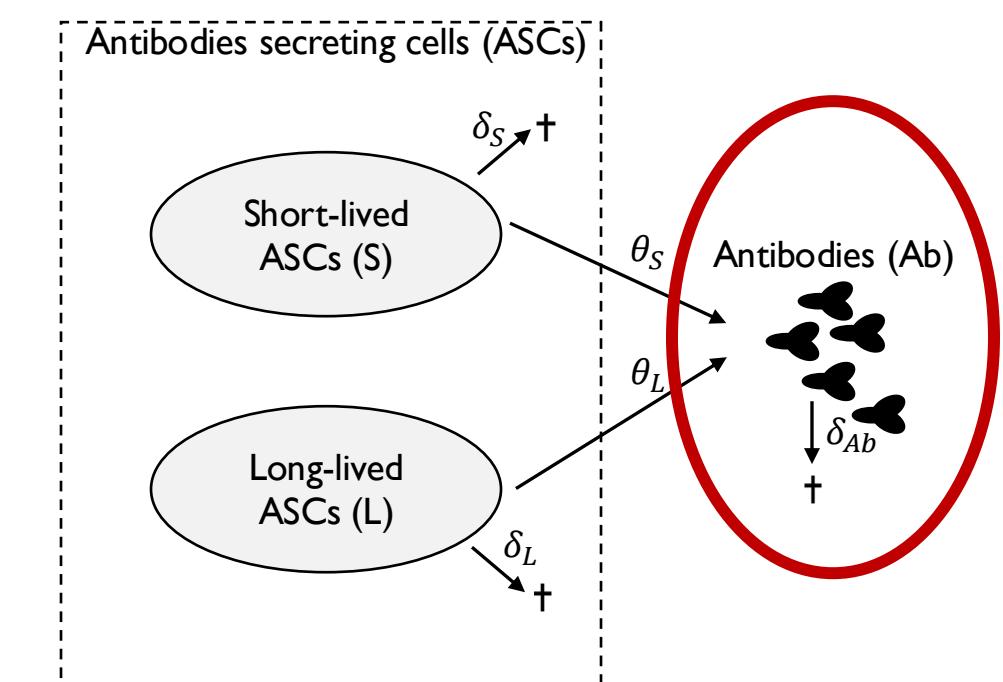
$$u_i \sim N(0, \Omega), \quad i \in \{1, \dots, N\}$$



□ **Observation model (Error model)**

$$Y_{ij} = Y(t_{ij}) = h(X(t_{ij}, \theta)) + g(X(t_{ij}, \theta), \Sigma) \epsilon_{ij}$$

$$\epsilon_{ij} \sim N(0, 1)$$



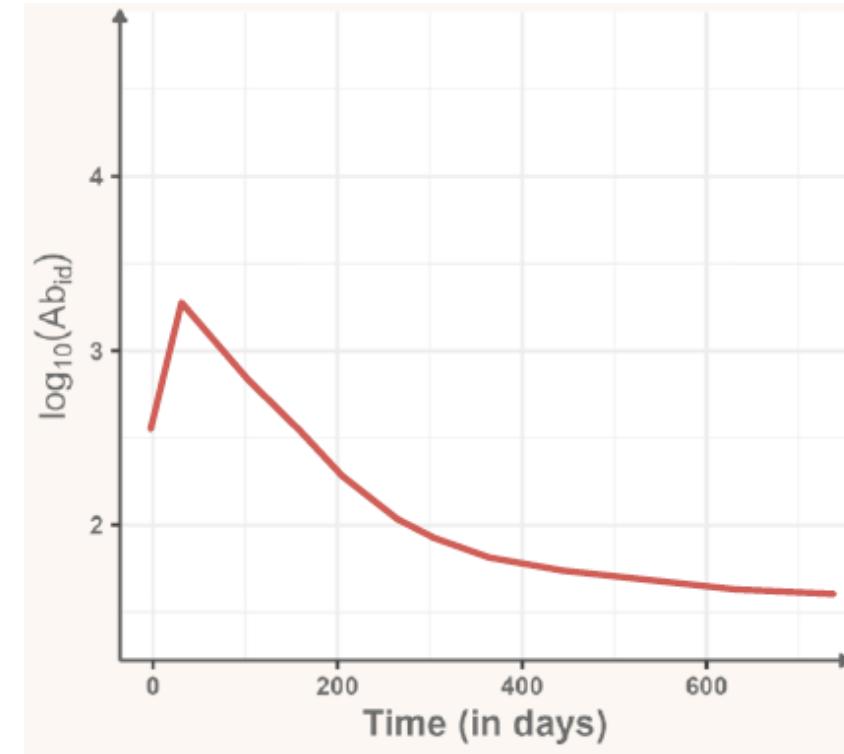
$$Y_{ij} = \log_{10}(Ab(\theta_i, t)) + \epsilon_{ij}$$

$$\epsilon_{ij} \sim N(0, \sigma)$$

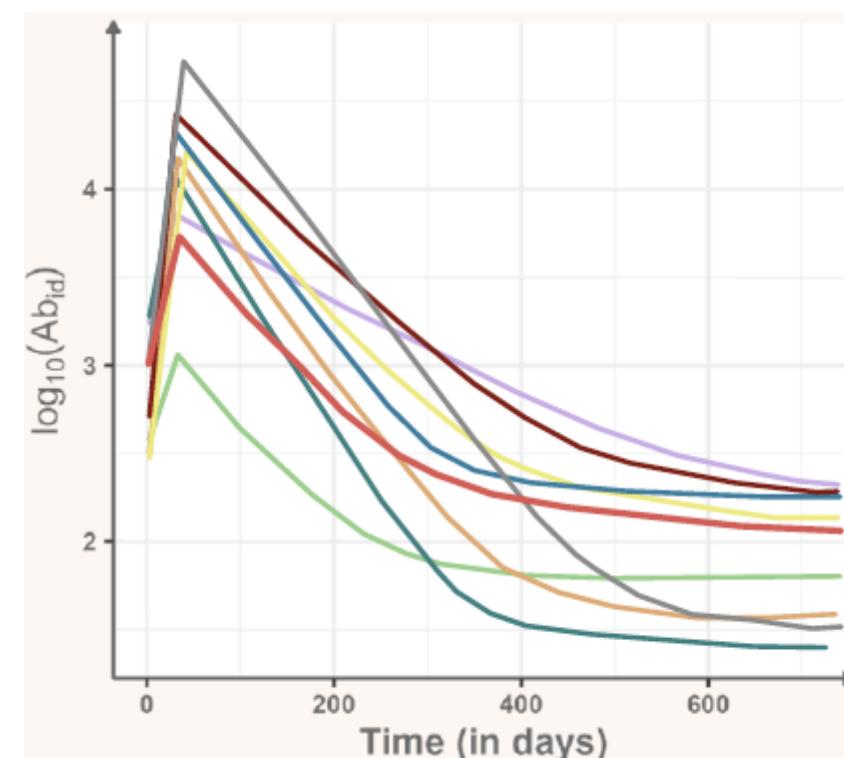
Mechanistic models – Population approach

□ Structural model (ODE-based)

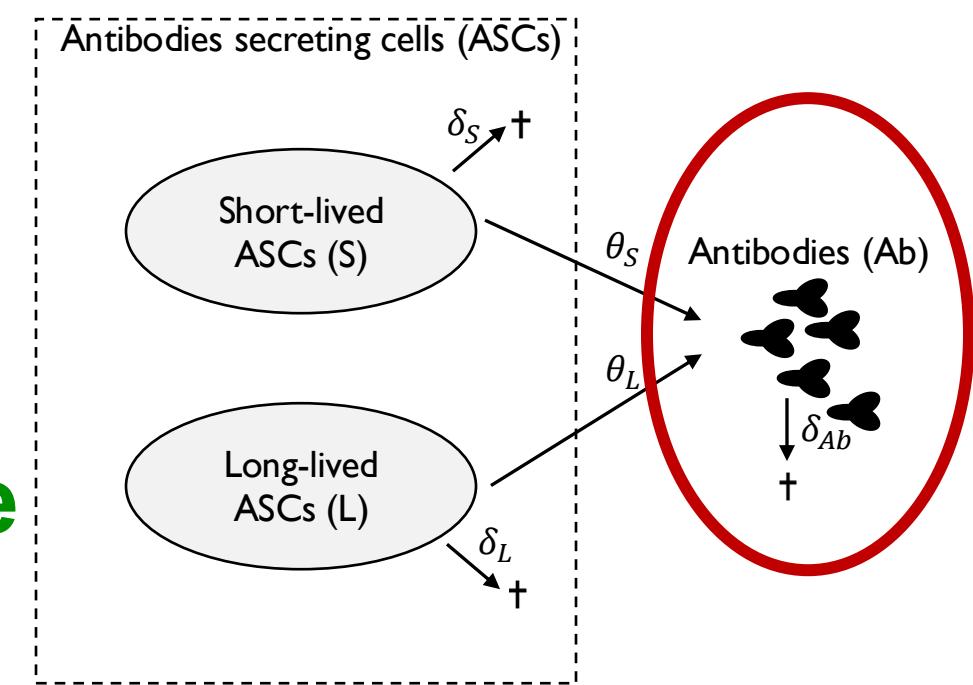
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...

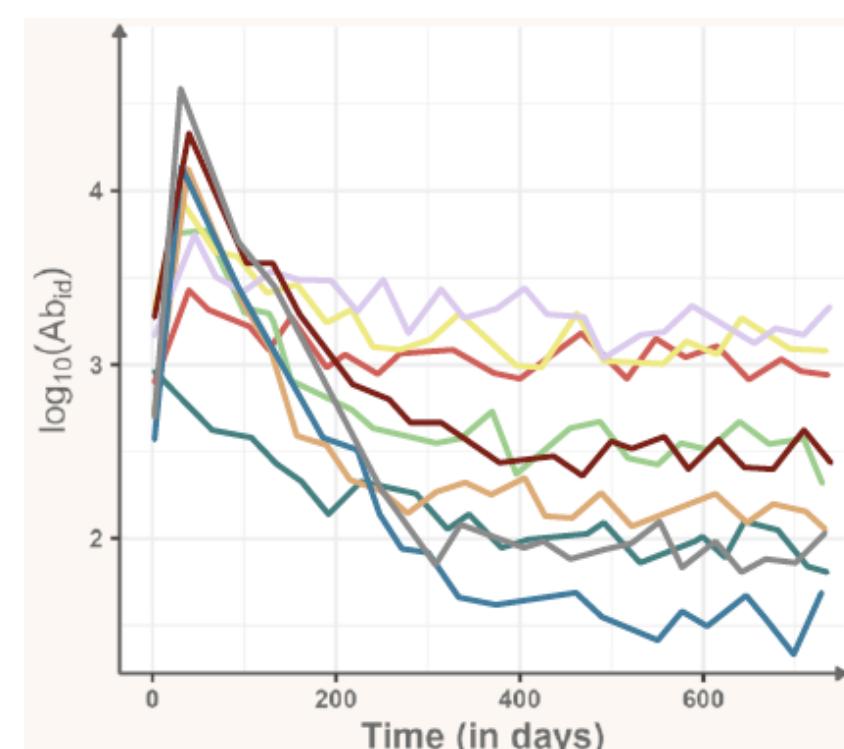


Inverse
→



$$Y_{ij} = \log_{10}(Ab(\theta_i, t)) + \epsilon_{ij}$$

$$\epsilon_{ij} \sim N(0, \sigma)$$



□ Statistical model (linear mixed effect model)

$$g(\theta_i(t)) = g(\theta_0) + \phi Z_i(t) + u_i$$

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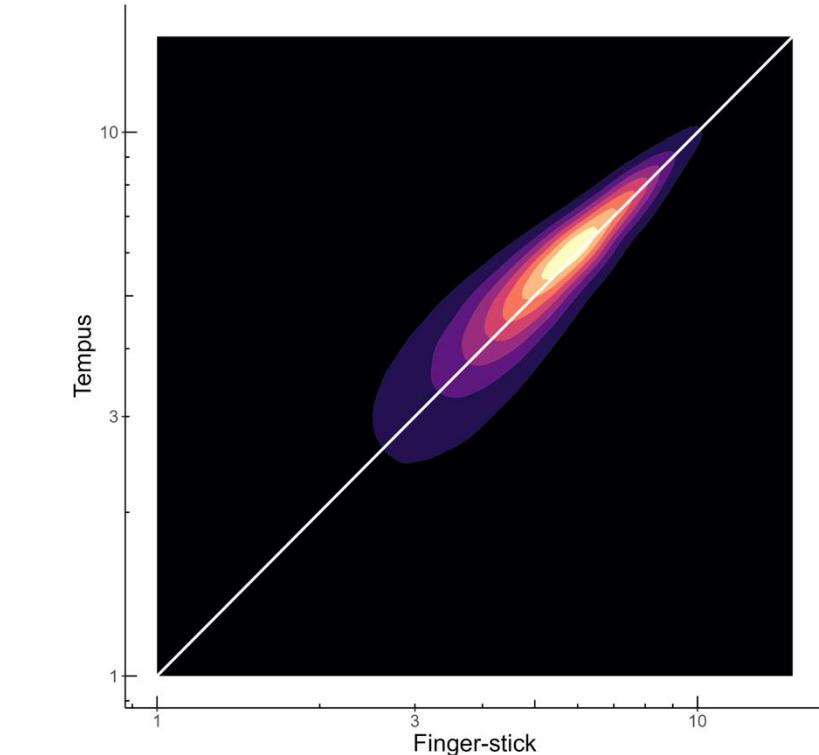
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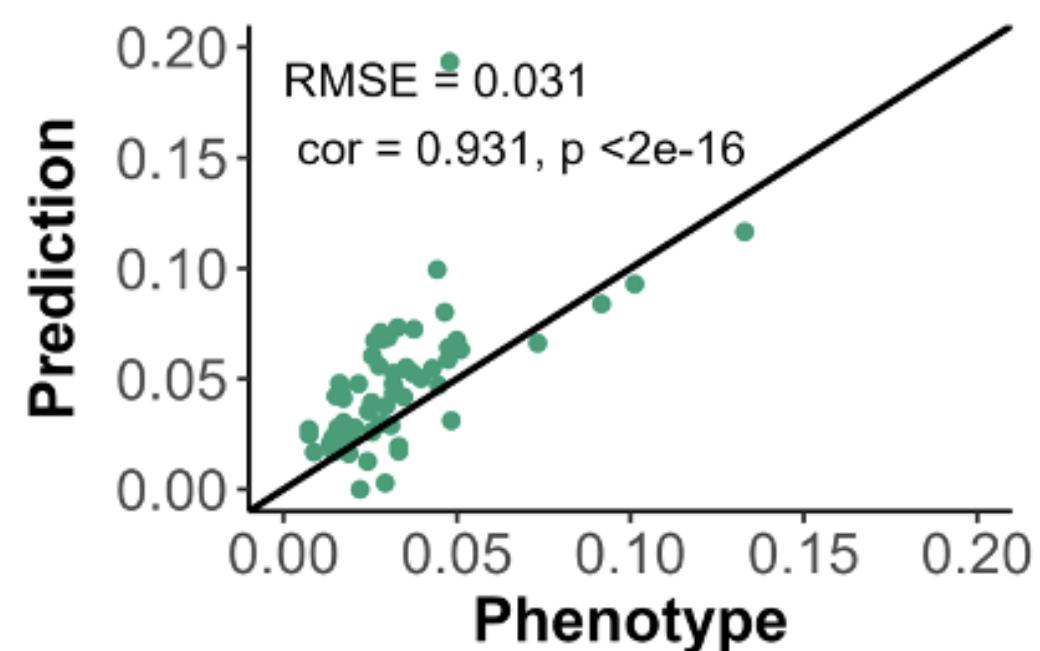
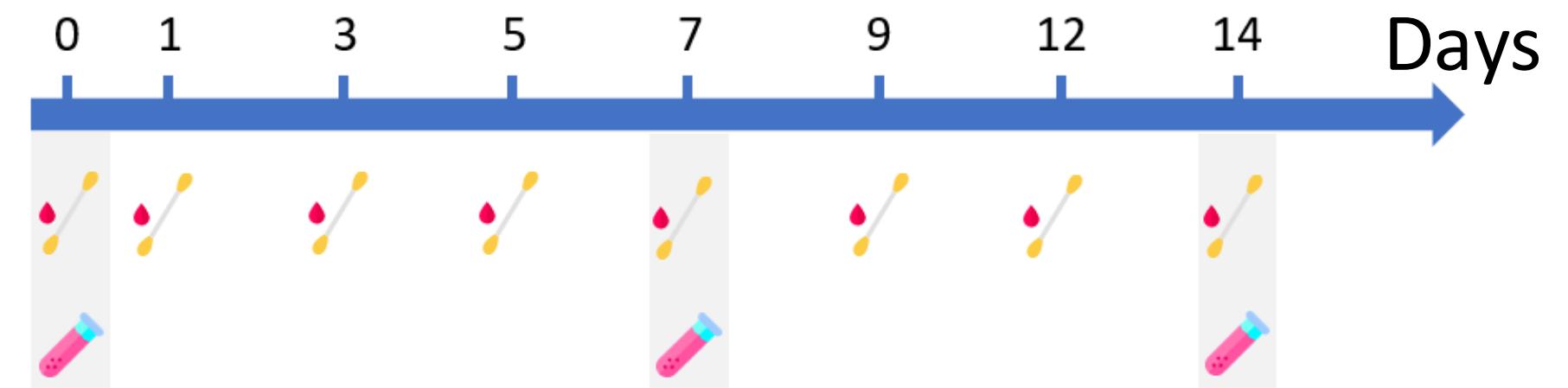
Rationale & Objective



High frequency transcriptomics (RNA-seq) from self collected finger-prick blood
Study: COVERAGE-Immuno



Good concordance between
the **whole-blood Tempus** and
Finger prick-test

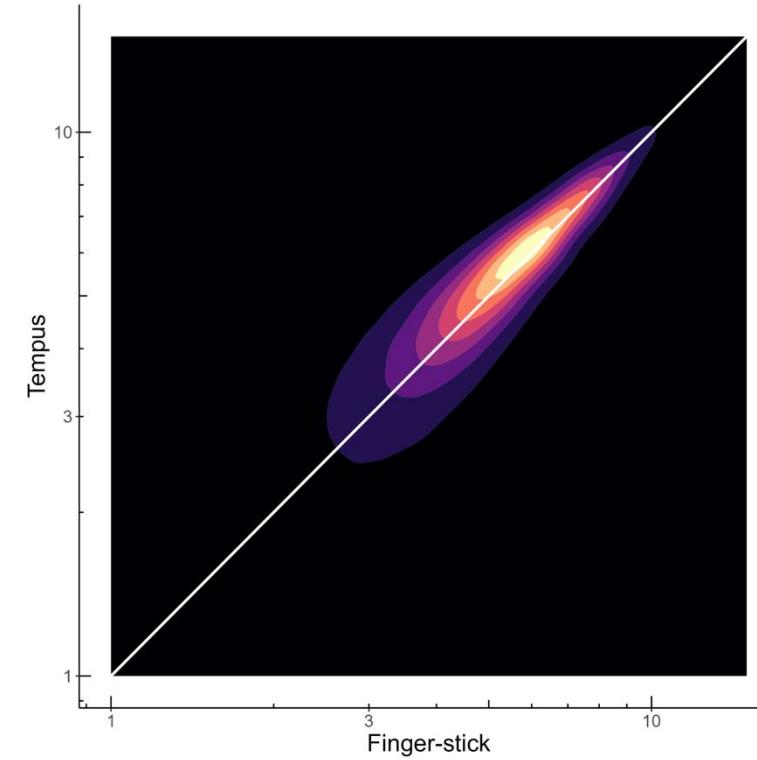


Reasonable concordance
between **cell abundance** by
ICS and from transcriptomic
data after deconvolution

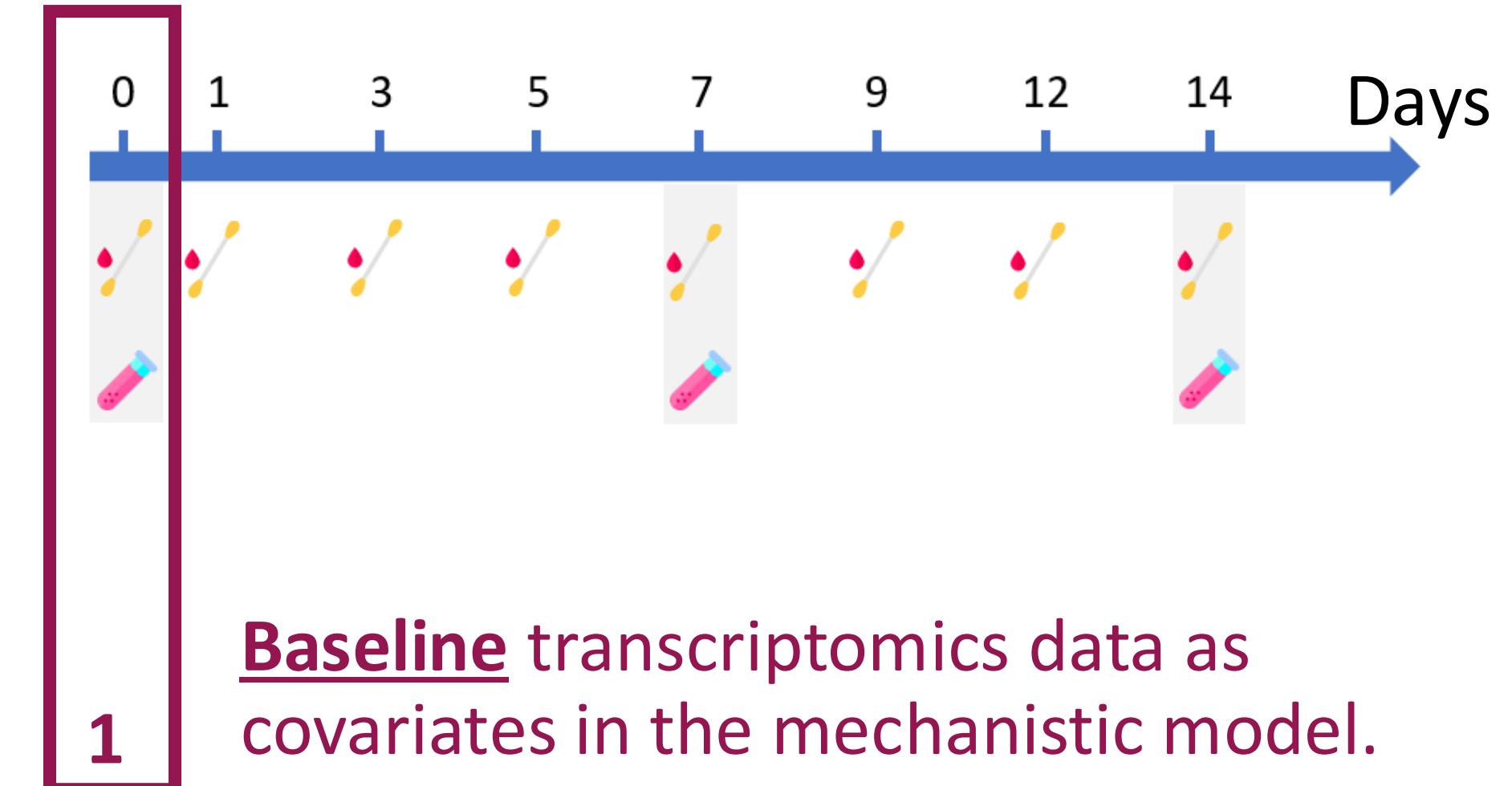
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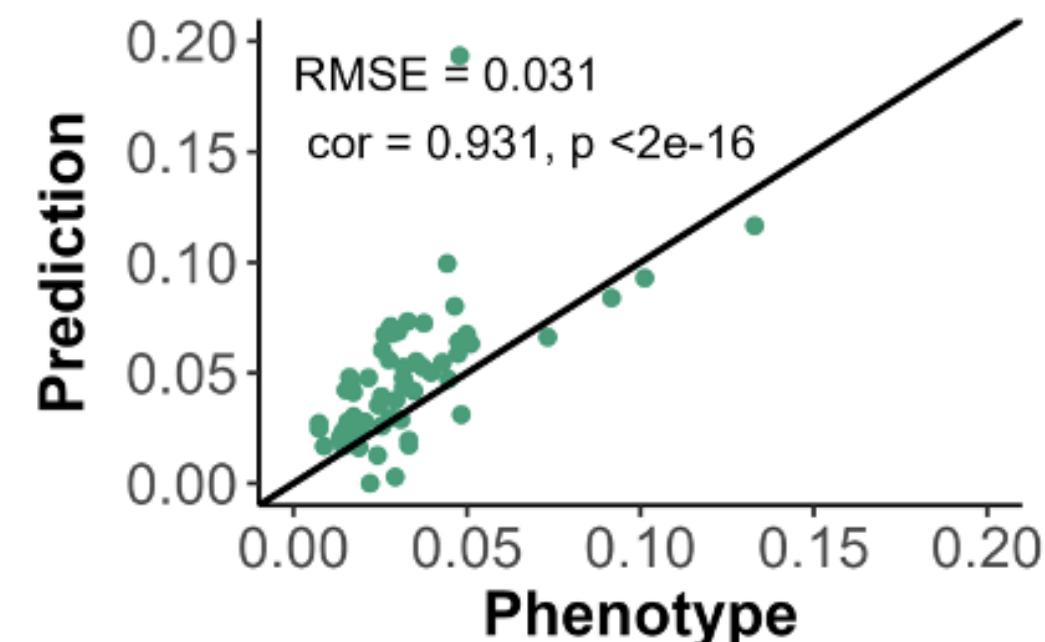
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Baseline transcriptomics data as
covariates in the mechanistic model.
R package Lasso-SAMBA

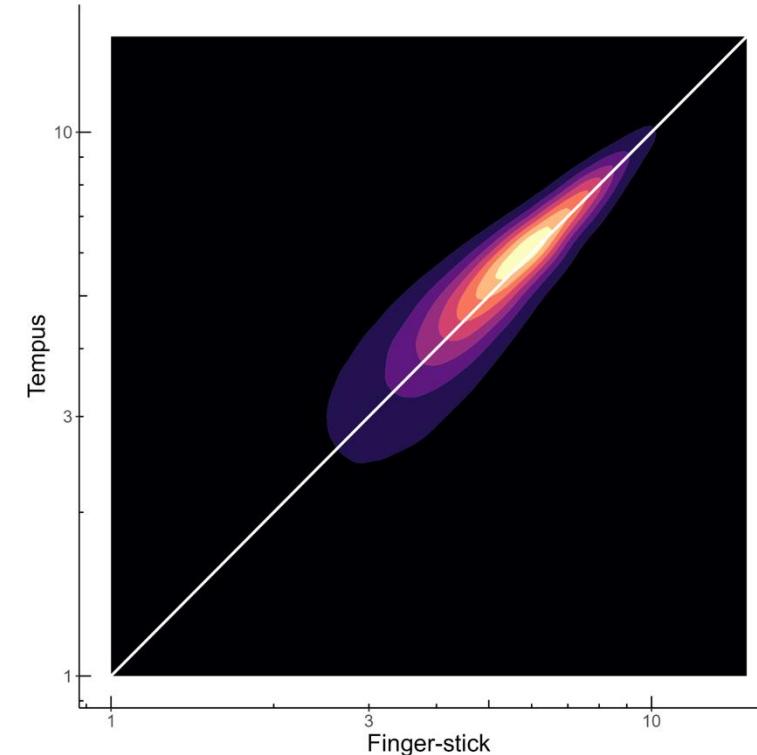


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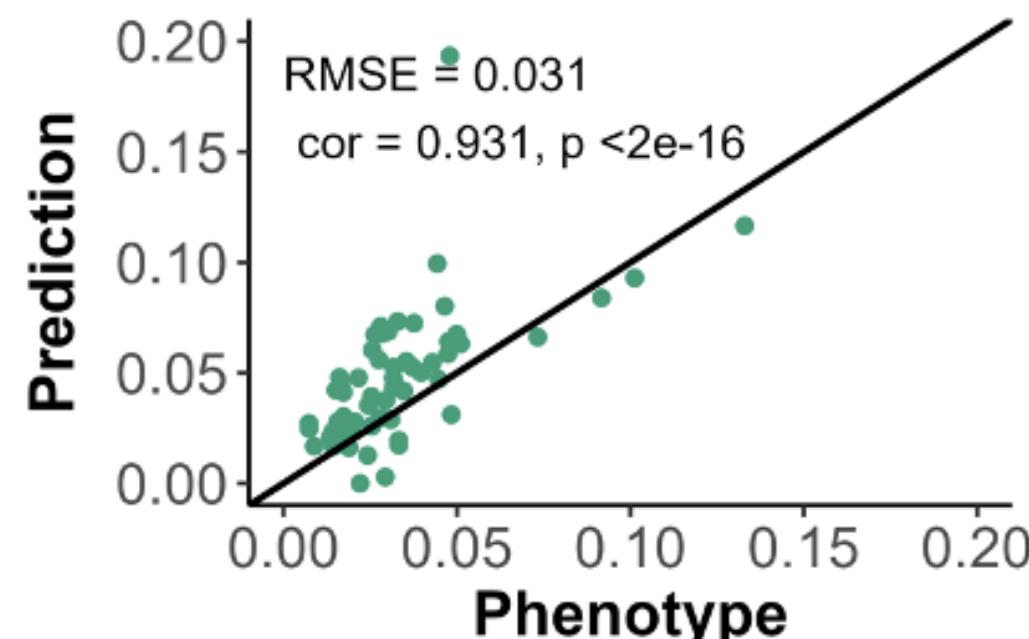
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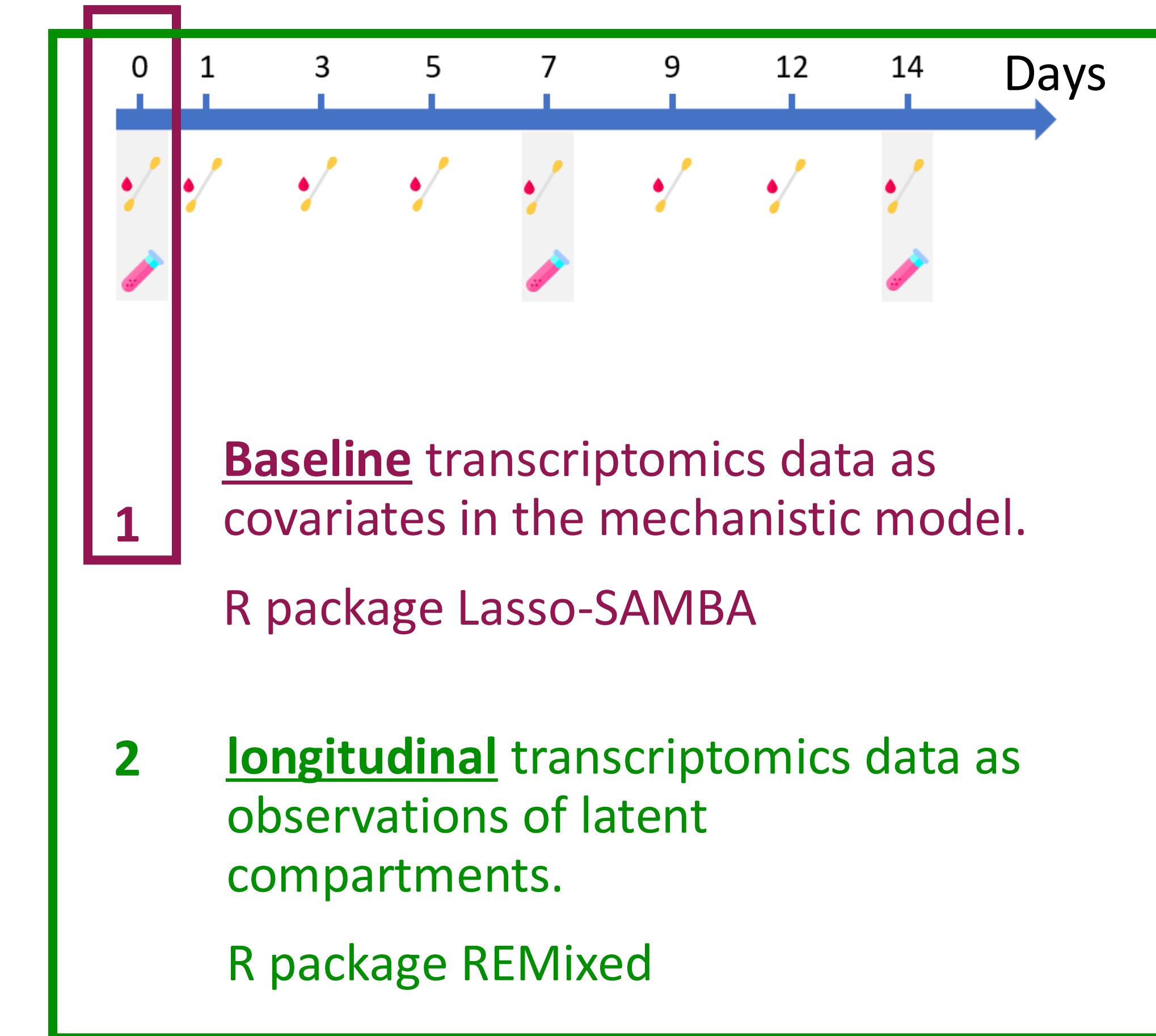
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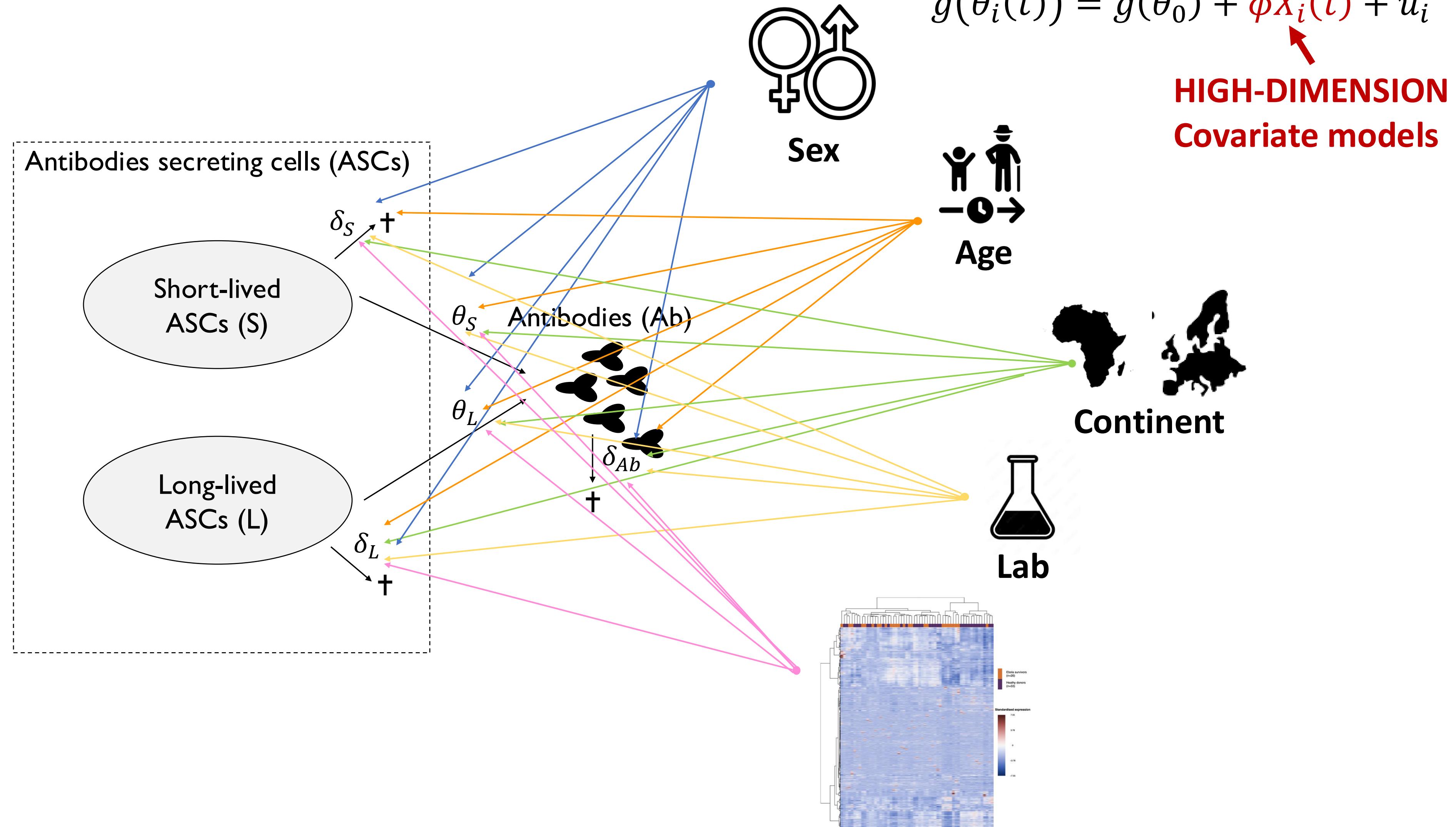


02

Using transcriptomic data as
explanatory covariates
(lasso-SAMBA Package)

Model Building strategy

Use baseline Gene expression as explanatory covariates

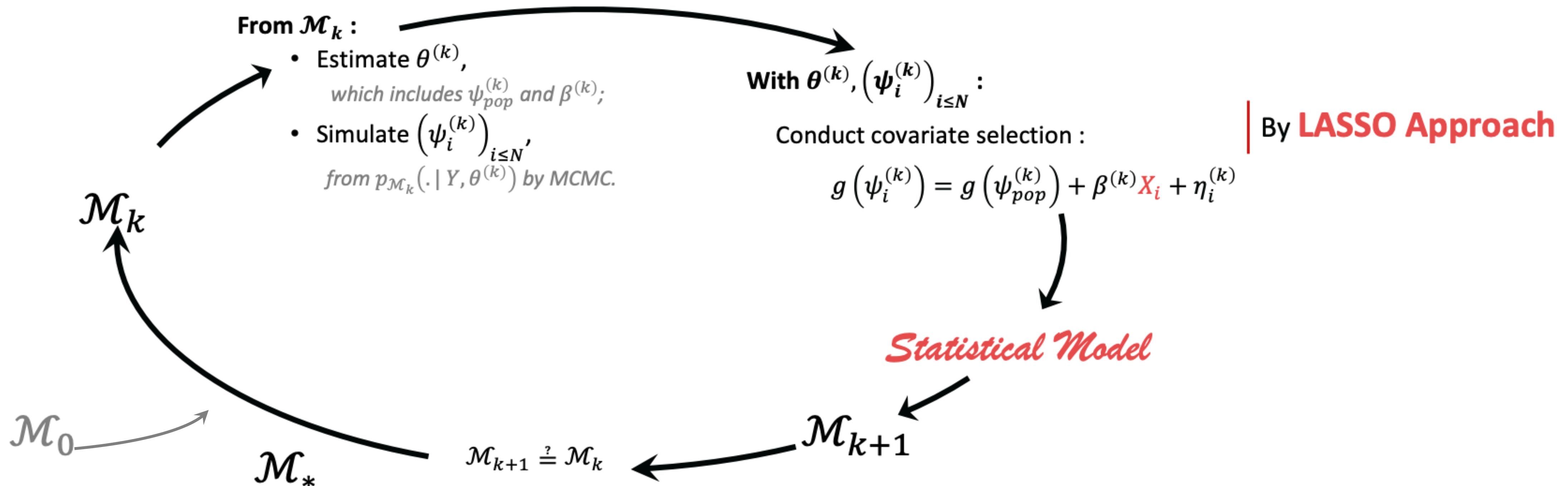


Method

Algorithms :

- SCM, *Stepwise Covariate Modeling* (Svensson and Jonsson, 2022) ;
- COSSAC, *COnditional Sampling use for Stepwise Approach based on Correlation tests* (Ayral and al. 2021) ;
- **SAMBA (Prague and Lavielle, 2022)**.

SAMBA, *Stochastic Approximation for Model Building Algorithm*, is an **iterative** algorithm, learning from a previous "worse" model in order to move towards a relevant model (Prague and Lavielle, 2022).



Simulations

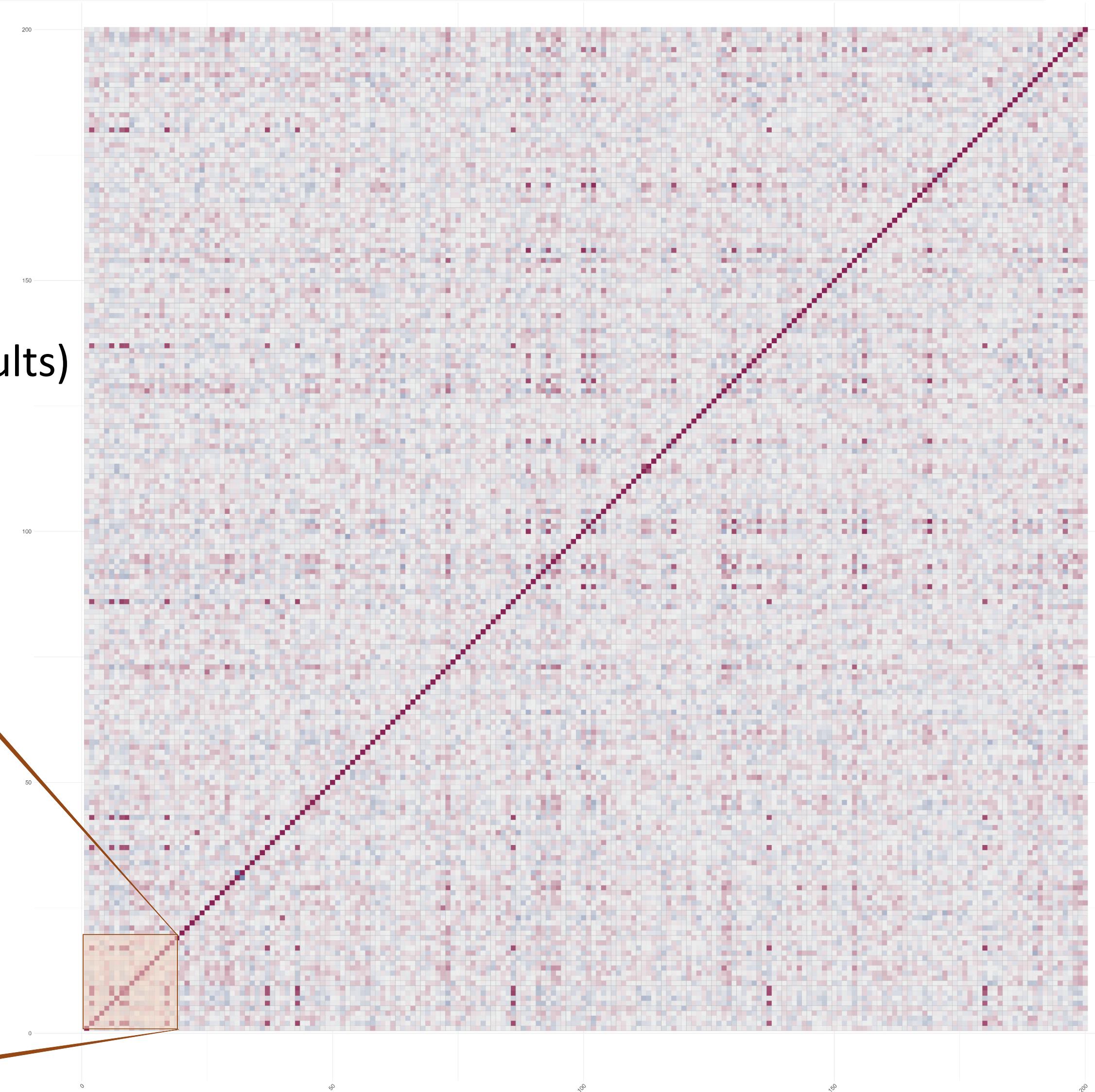
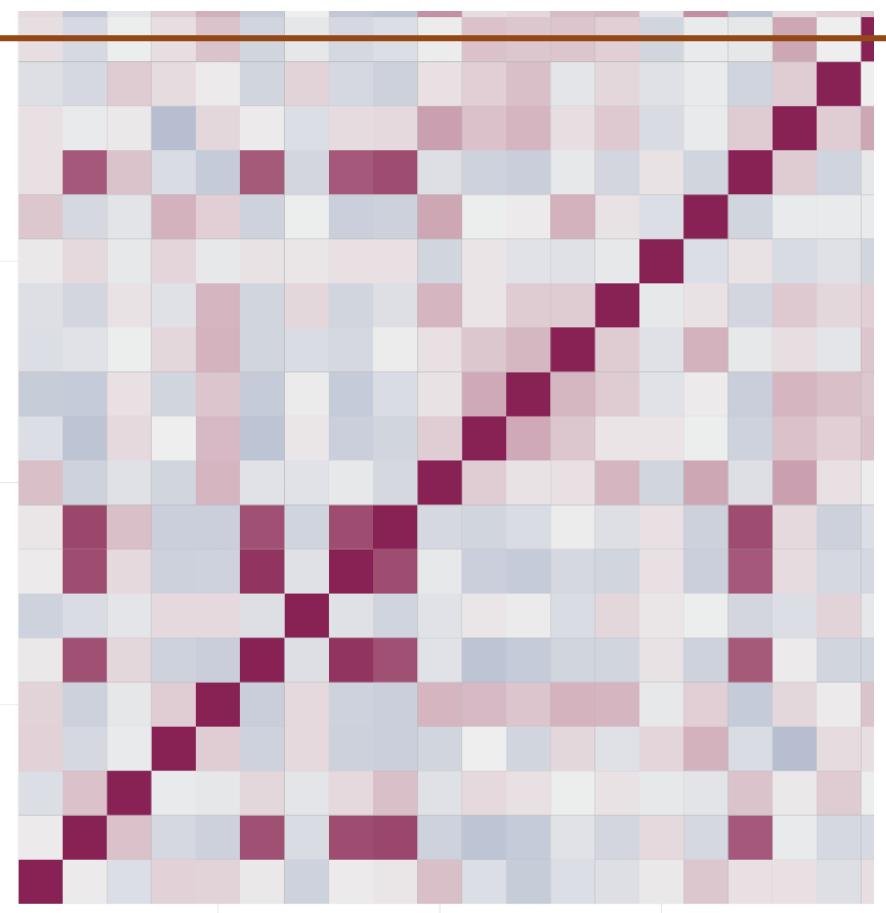
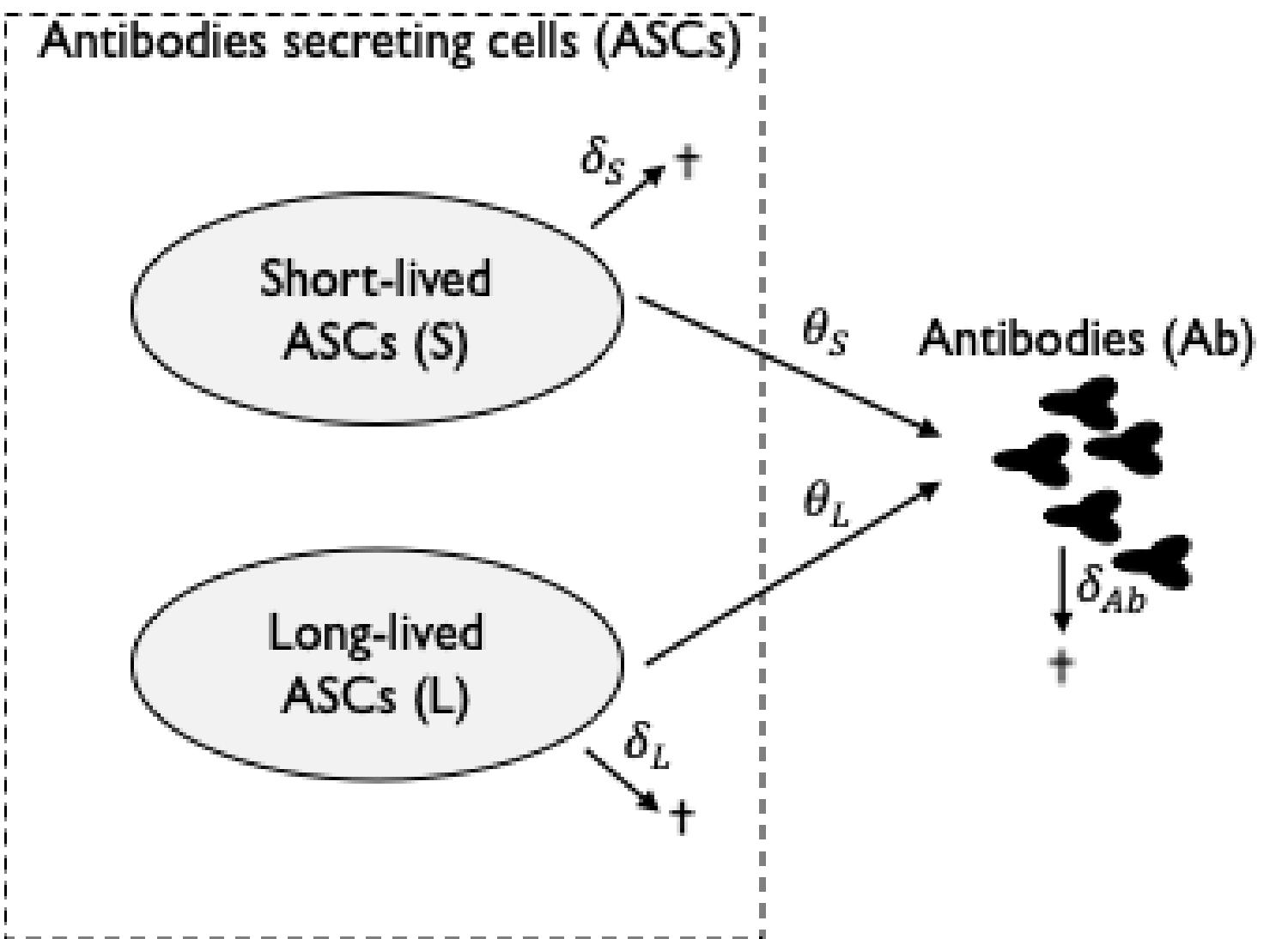
Covariates effects :

- $AGE \sim \mathcal{N}(35, 4^2)$ on φ_S ,
- $G_1 \sim \mathcal{N}(0, 1)$ on φ_L
- $G_2 \sim \mathcal{N}(0, 1)$ on δ_{Ab}

$P = 200$ gaussian **correlated** covariates ($P=1000$ similar results)

$R = 100$ replicates,

$N = 100$ individuals ($N=20$ similar results).



Simulations

Parameter-Covariate link	Selected in the final model	NOT selected in the final model
In the generation model	True Positive (TP)	False Negative (FN)
NOT in the generation model	False Positive (FP)	True Negative (TN)

$$\text{False Discovery Rate : } FDR = \frac{FP}{TP + FP}$$

$$\text{False Negative Rate : } FNR = \frac{FN}{TN + FN}$$

$$\text{F1-score : } F1_{score} = \frac{2TP}{2TP + FN + FP}$$

Simulations

Error Rate Comparison Table

Rate	Median	Confidence Interval (quantiles 95%)
False Discovery Rate : - stepAIC with stat. test - Lasso : $E[FDR] < 10\%$	72.7% 0.0%	[50.0%;83.8%] [0.0%;25.0%]
False Negative Rate : - stepAIC with stat. test - Lasso : $E[FDR] < 10\%$	0.0% 0.0%	[0.0%;0.0%] [0.0%;0.0%]
F1 score : - stepAIC with stat. test - Lasso : $E[FDR] < 10\%$	42.9% 100.0%	[27.9%;66.7%] [85.7%;100.0%]

• Final Final model without any False Negatives :
- stepAIC with stat. test : 99%
- Lasso : $E[FDR] < 10\% : 100\%$

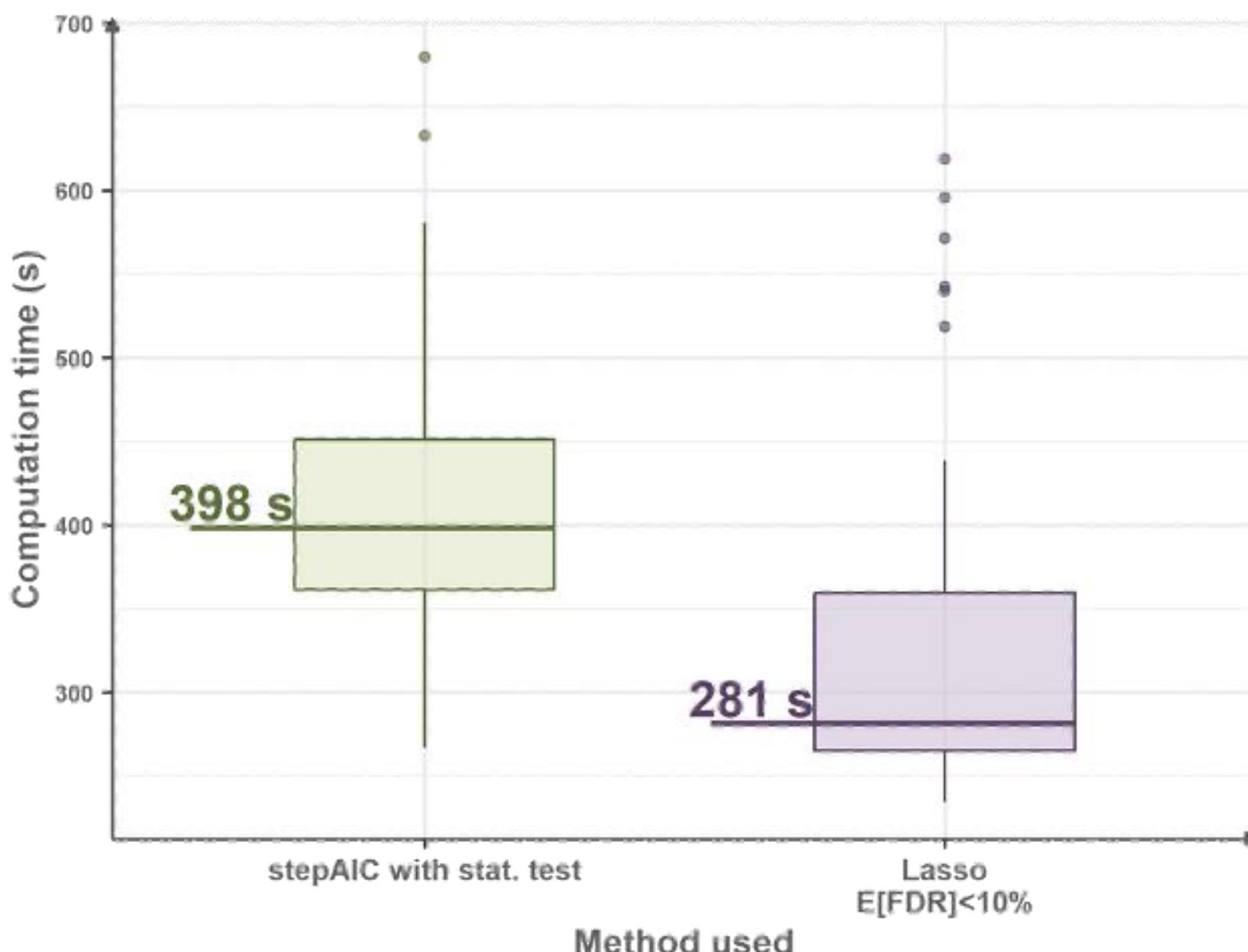
• Final model is the true one :
- stepAIC with stat. test : 0%
- Lasso : $E[FDR] < 10\% : 81\%$

Covariates presence in final model with parameters link

Among 100 simulated datasets of humoral immune response to prime-boost of Ad26.ZEBOV/MVA-BN-FILO vaccine against Ebola for 100 individuals, with 200 gaussian correlated covariates.

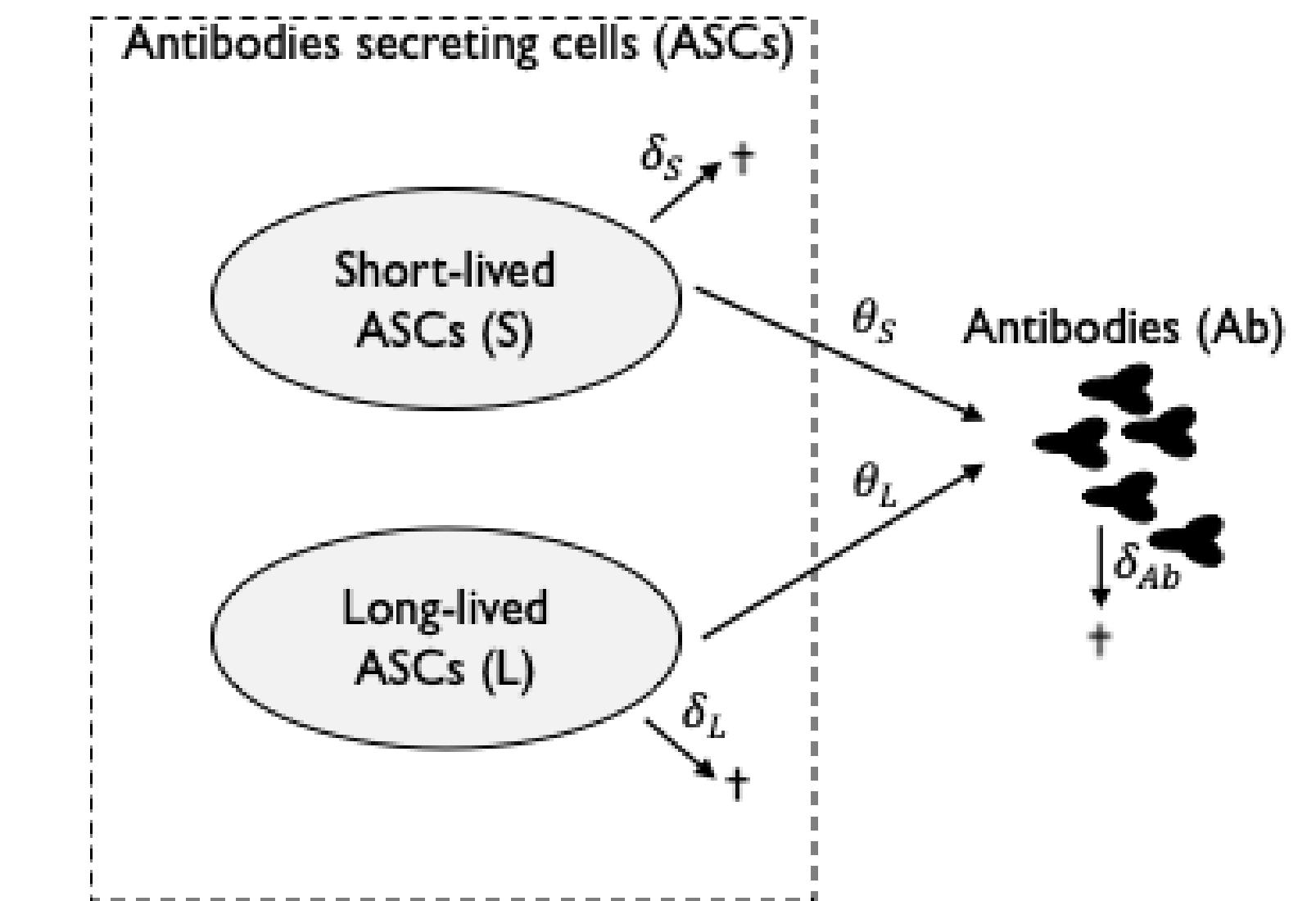
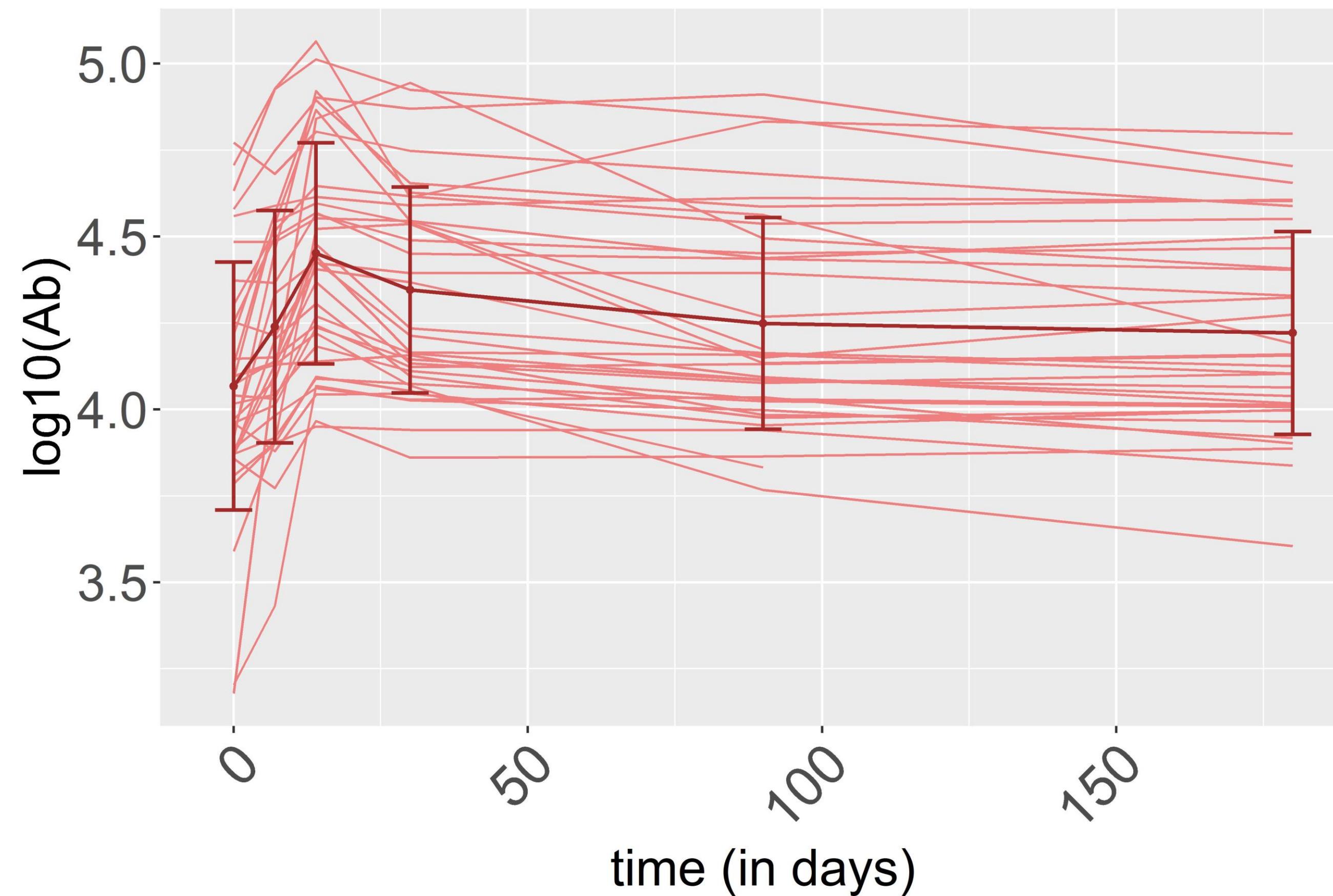
Computation Time Comparison

Among 100 simulated datasets of humoral immune response to prime-boost of Ad26.ZEBOV/MVA-BN-FILO vaccine against Ebola for 100 individuals, with 200 gaussian correlated covariates.



Application to vaccine study : VZV

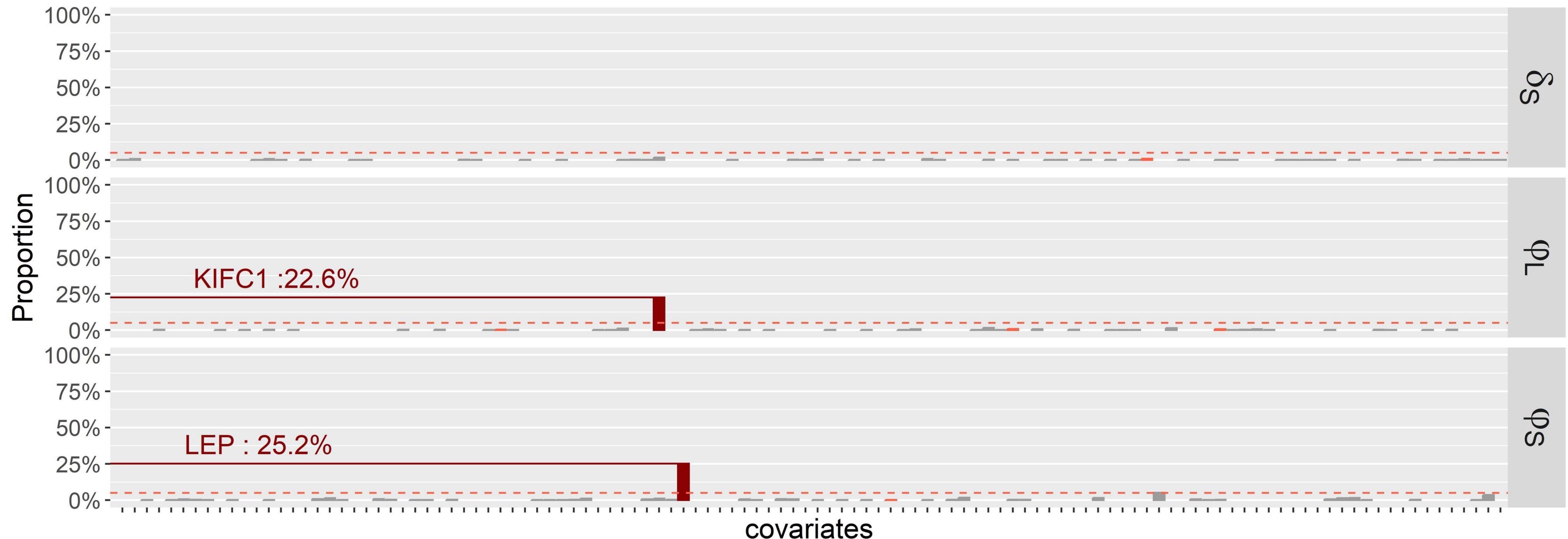
- Clinical study of immune response to vaccination against the **Varicella-Zoster Virus (VZV)**
- Gene expression and antibody response data following immunization with ZOSTAVAX, a live attenuated vaccine.
- 35 adult volunteers, 6 datapoints at day 0, 7, 14, 30, 90, and 180



Application to vaccine study : VZV

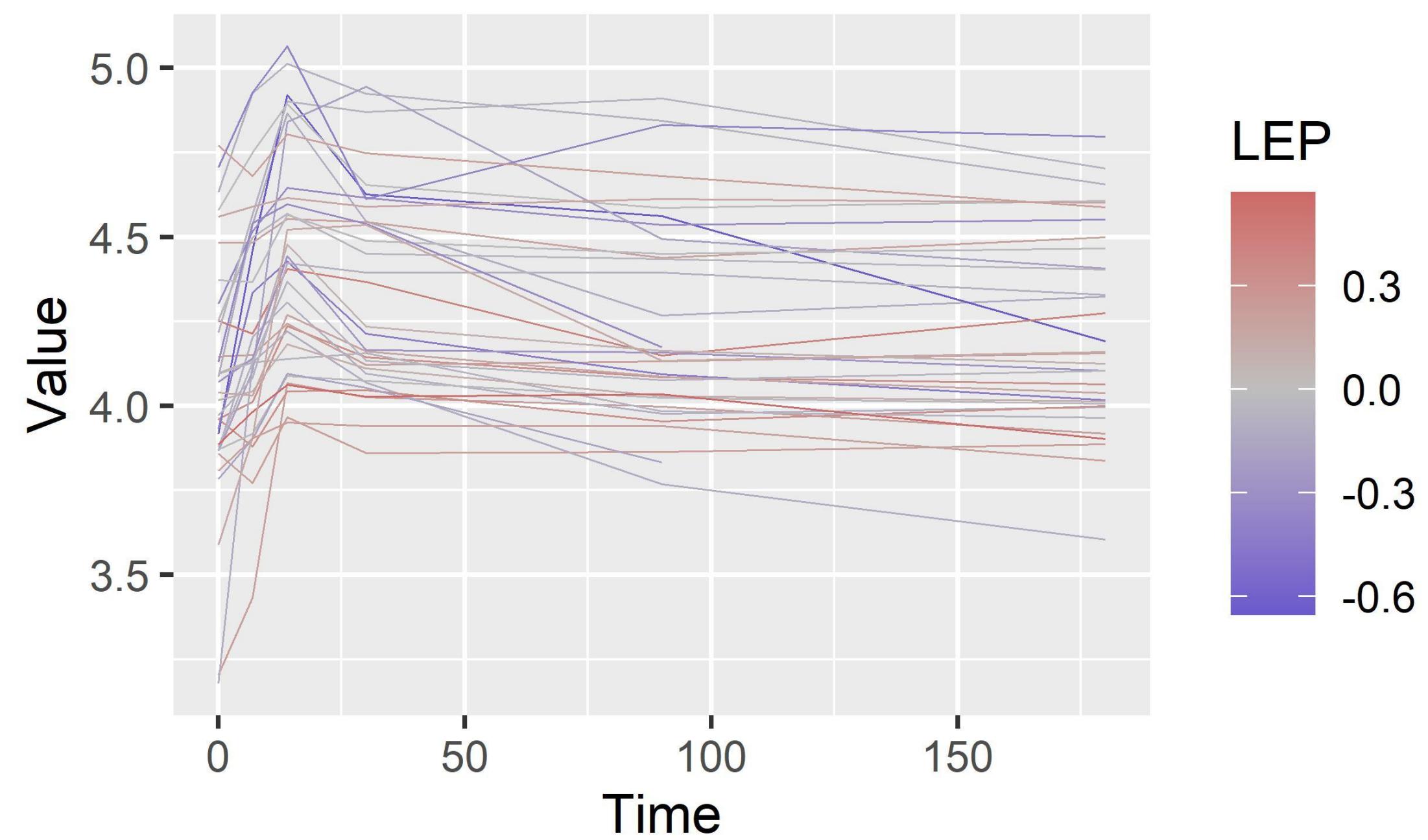
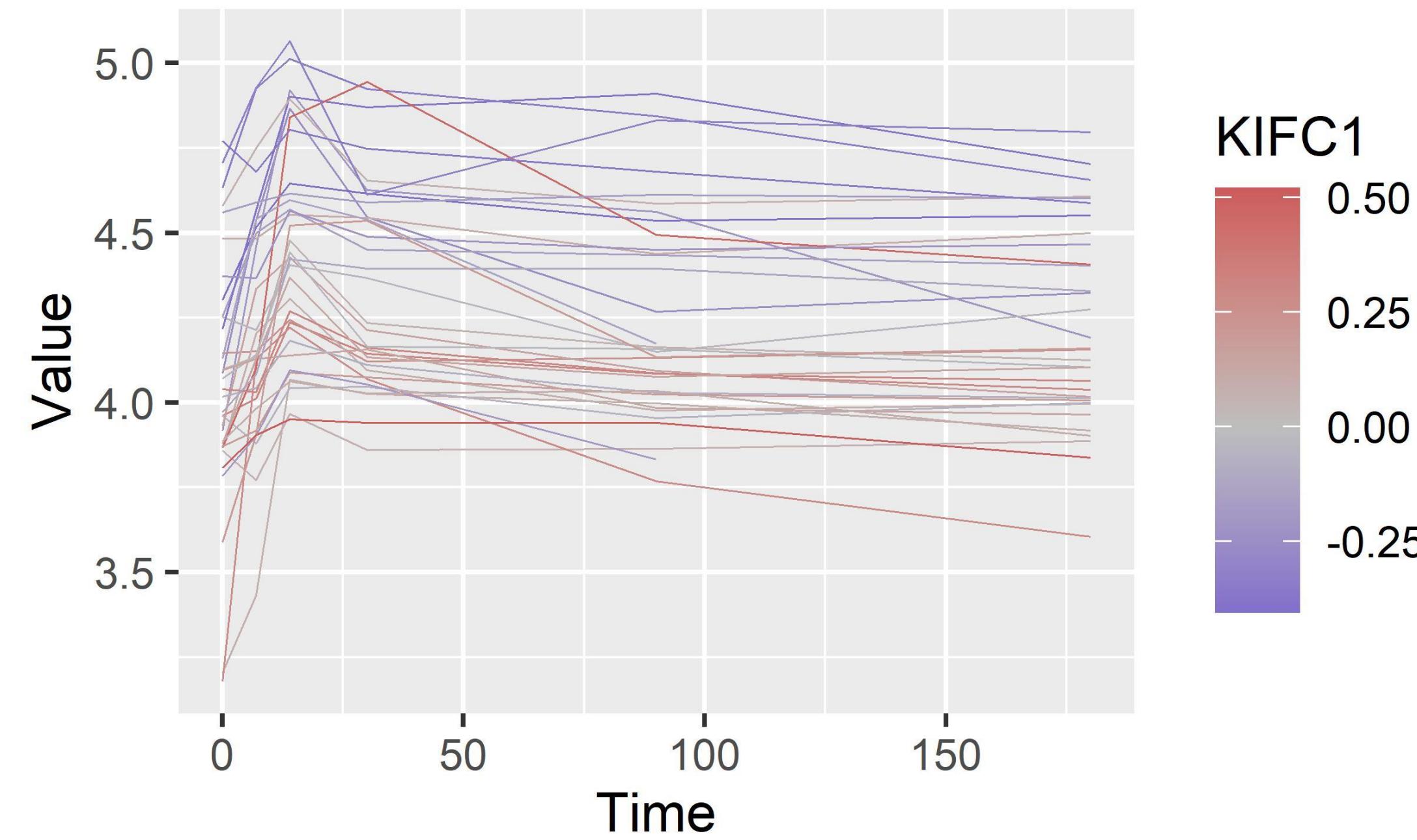
- 10,086 profiled genes
- a subset of 784 protein-coding genes was selected based on functional annotation with roles in
 - Interferon signaling,
 - Type I Interferon response,
 - Neutrophil activation,
 - Inflammation,
 - Cytokine/chemokine activity,
 - and Cell cycle regulation

ASSOCIATED GENES : KIFC1 & LEP



Interpretation

- The LEP gene is a player in several common biological pathways involved in the immune response such as JAK-STAT or NFkB.
- KIFC1 is involved in cell proliferation and therefore is not specific of immune response. KIFC1 also has the ability to promote stable mitotic spindle formation during early B cell development where centriole duplication is frequent but must be tightly regulated



Francisco et al. Obesity, Fat Mass and Immune System: Role for Leptin. *Frontiers in Physiology*. 2018

Lam et al. Role of leptin in immunity. *Cell Mol Immunol*. 2007

Lucanus et al. Kinesin superfamily: roles in breast cancer, patient prognosis and therapeutics. *Oncogene*. 2018

Wu et al. An integrative pan-cancer analysis of kinesin family member C1 (KIFC1) in human tumors. *Biomedicines*. 2022

Hagan et al. Transcriptional atlas of the human immune response to 13 vaccines reveals predictor of vaccine-induced Ab responses. *Nature Immunology*. 2022

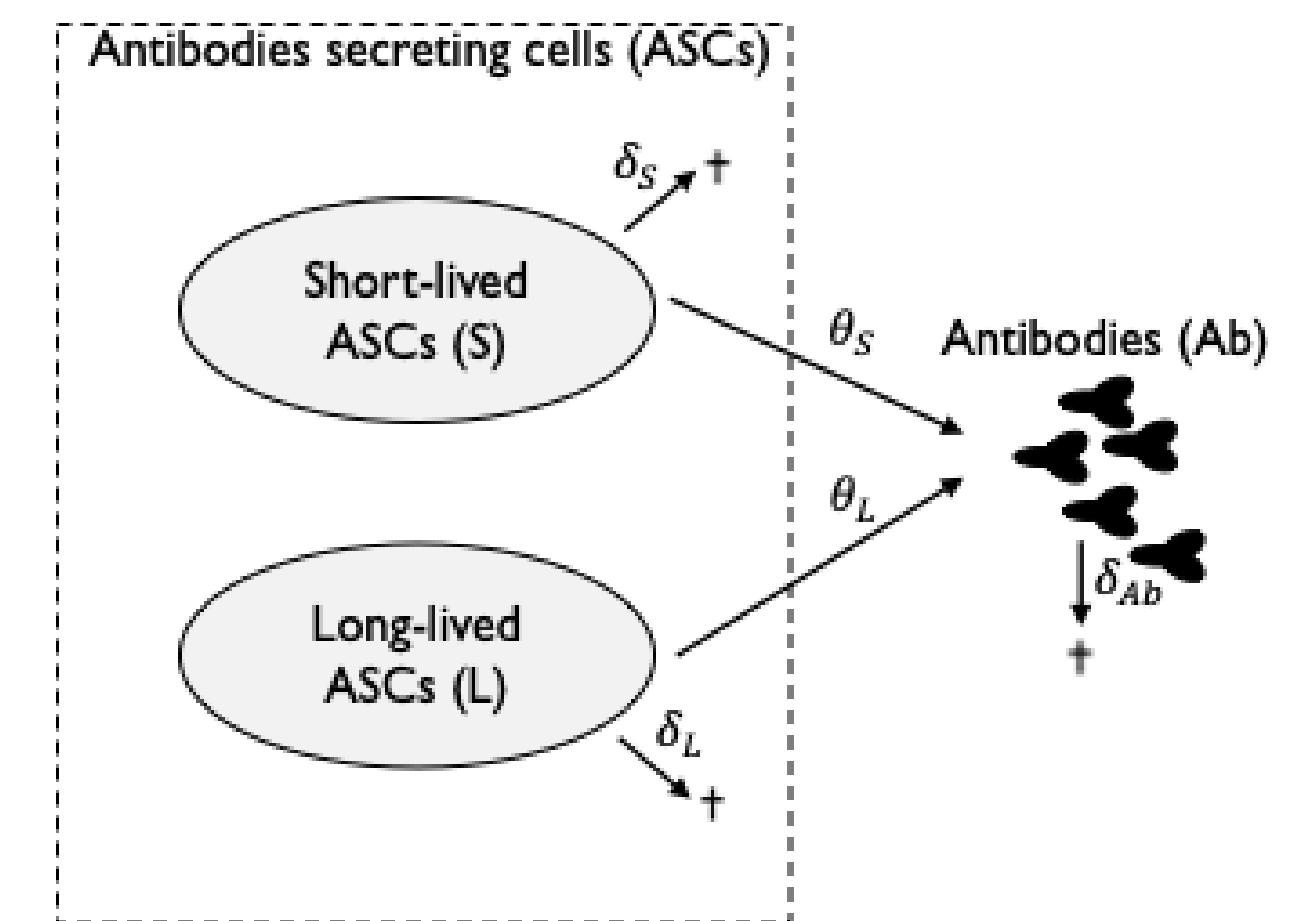
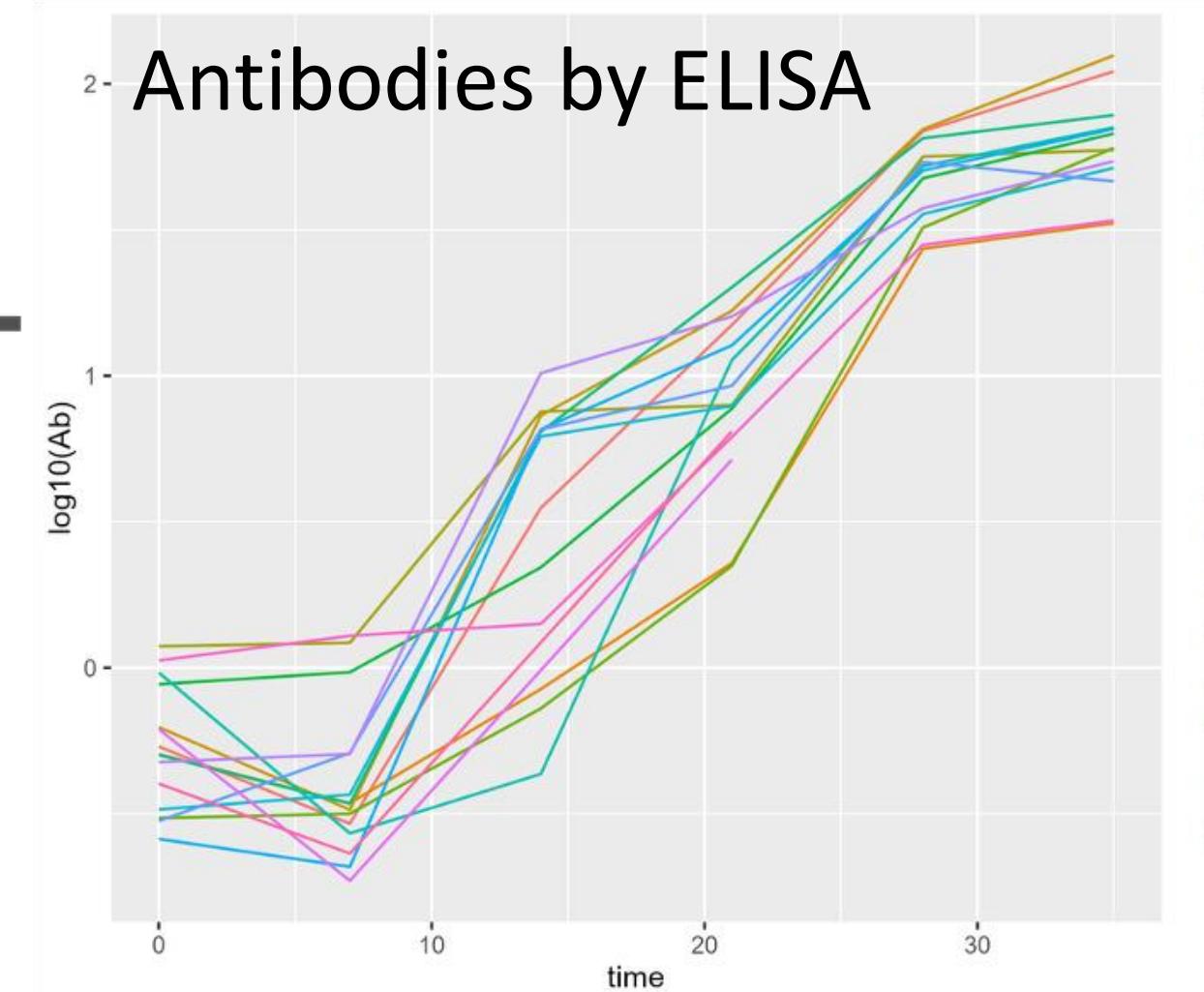
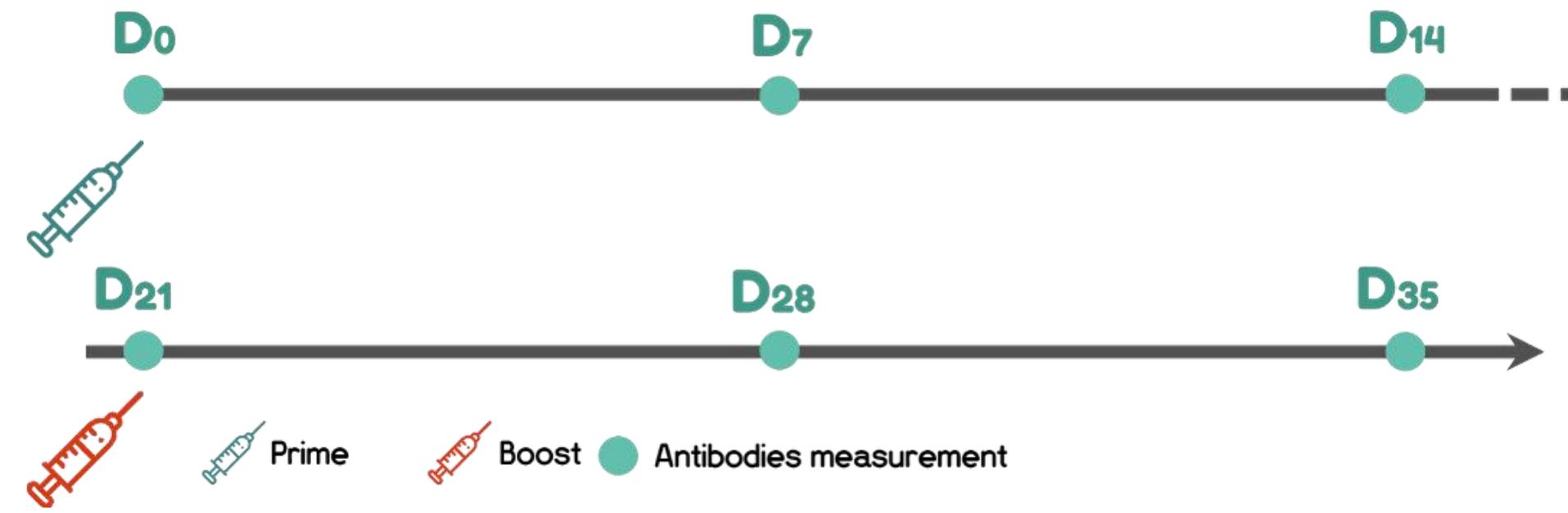
03

Toward high dimension
mechanistic models using latent
class models
(REmixed Package)

Motivating study

Evaluation of COVID-19 vaccine

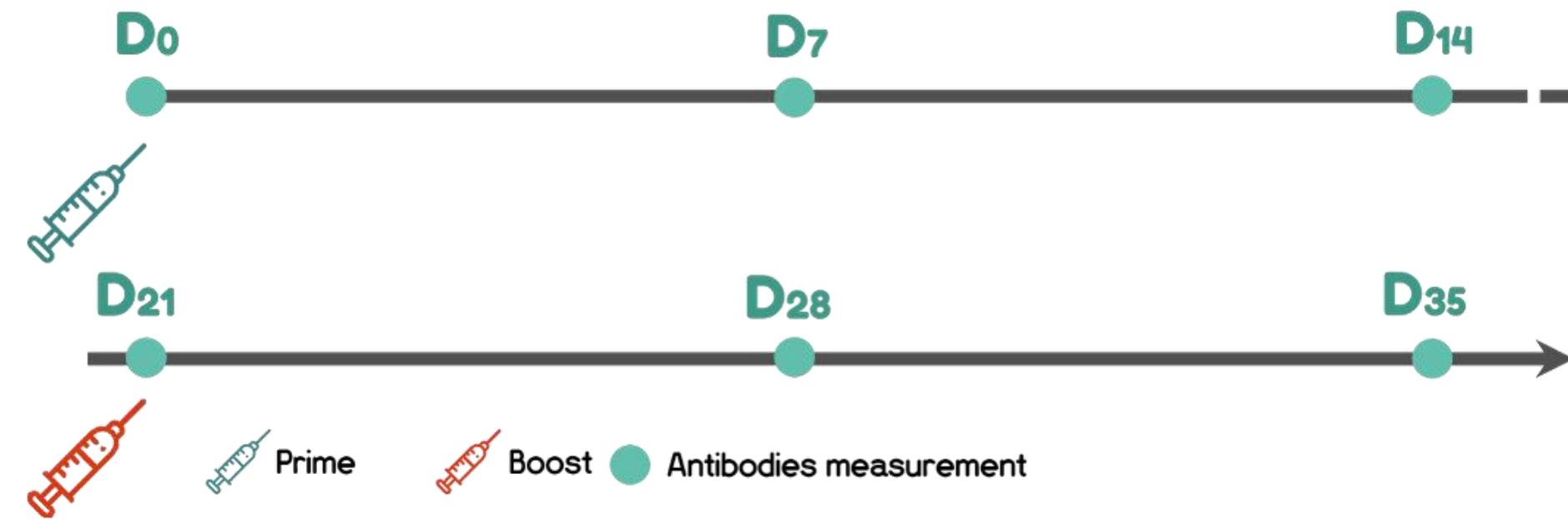
15 adults receiving COVID-19 Pfizer vaccine



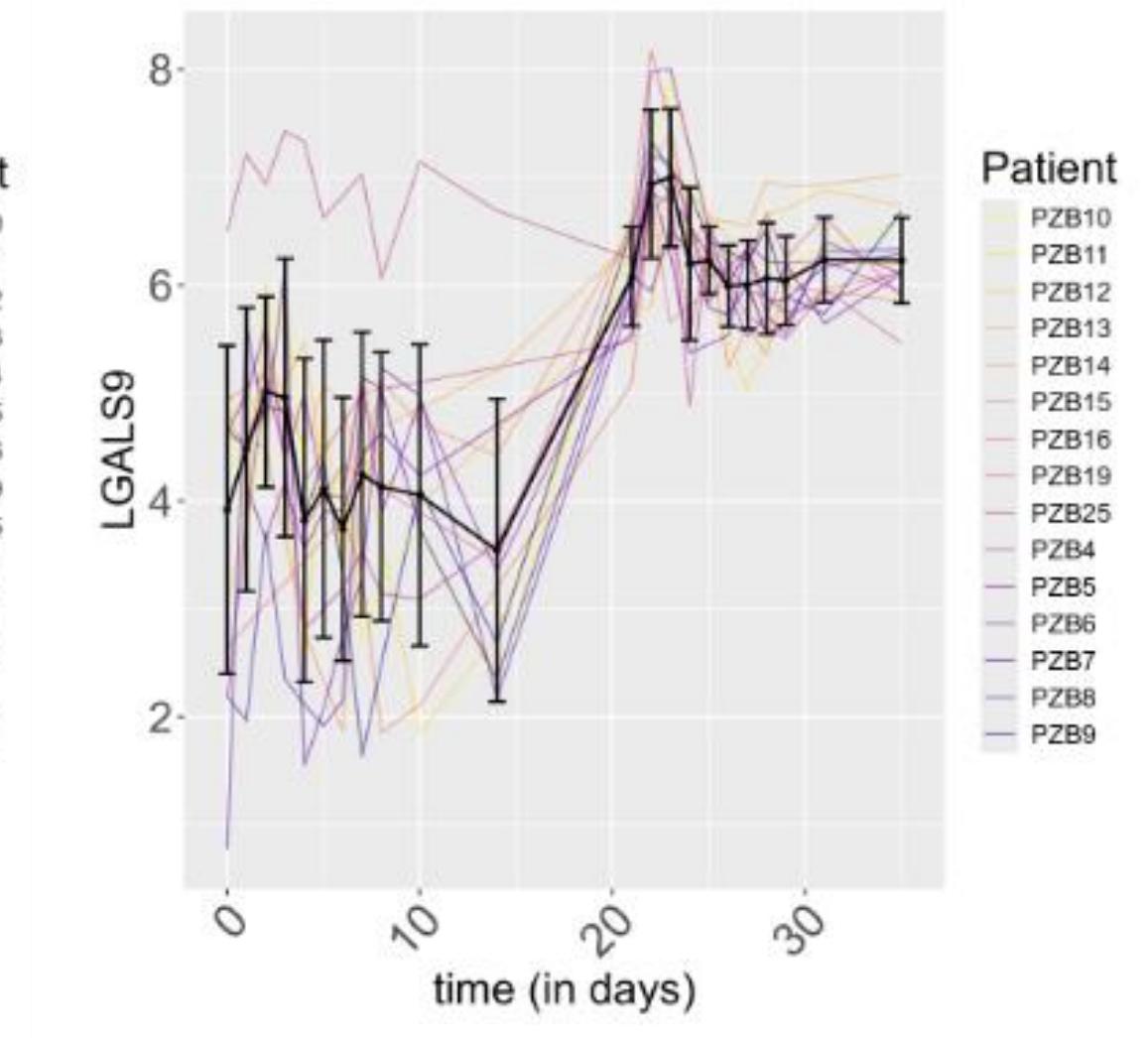
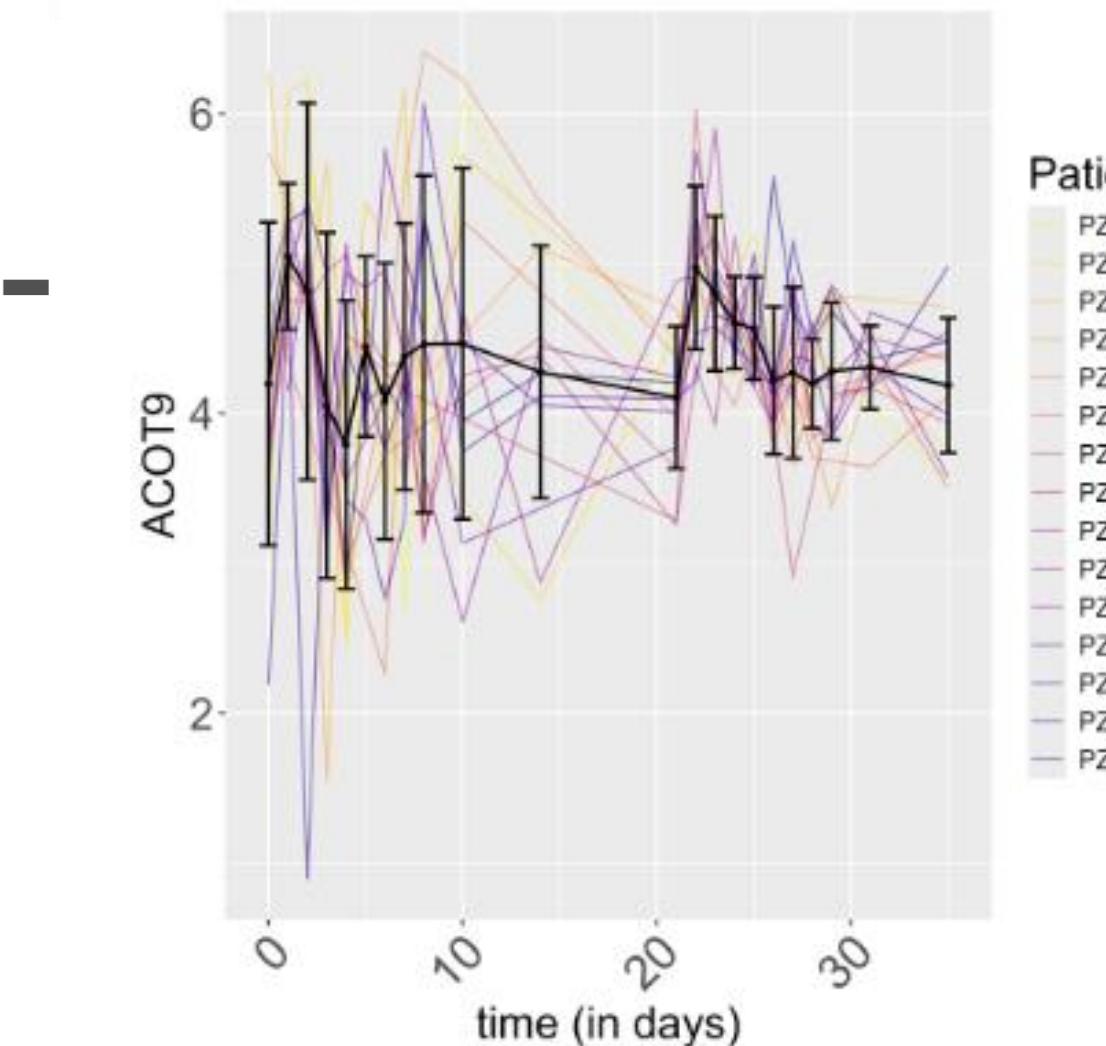
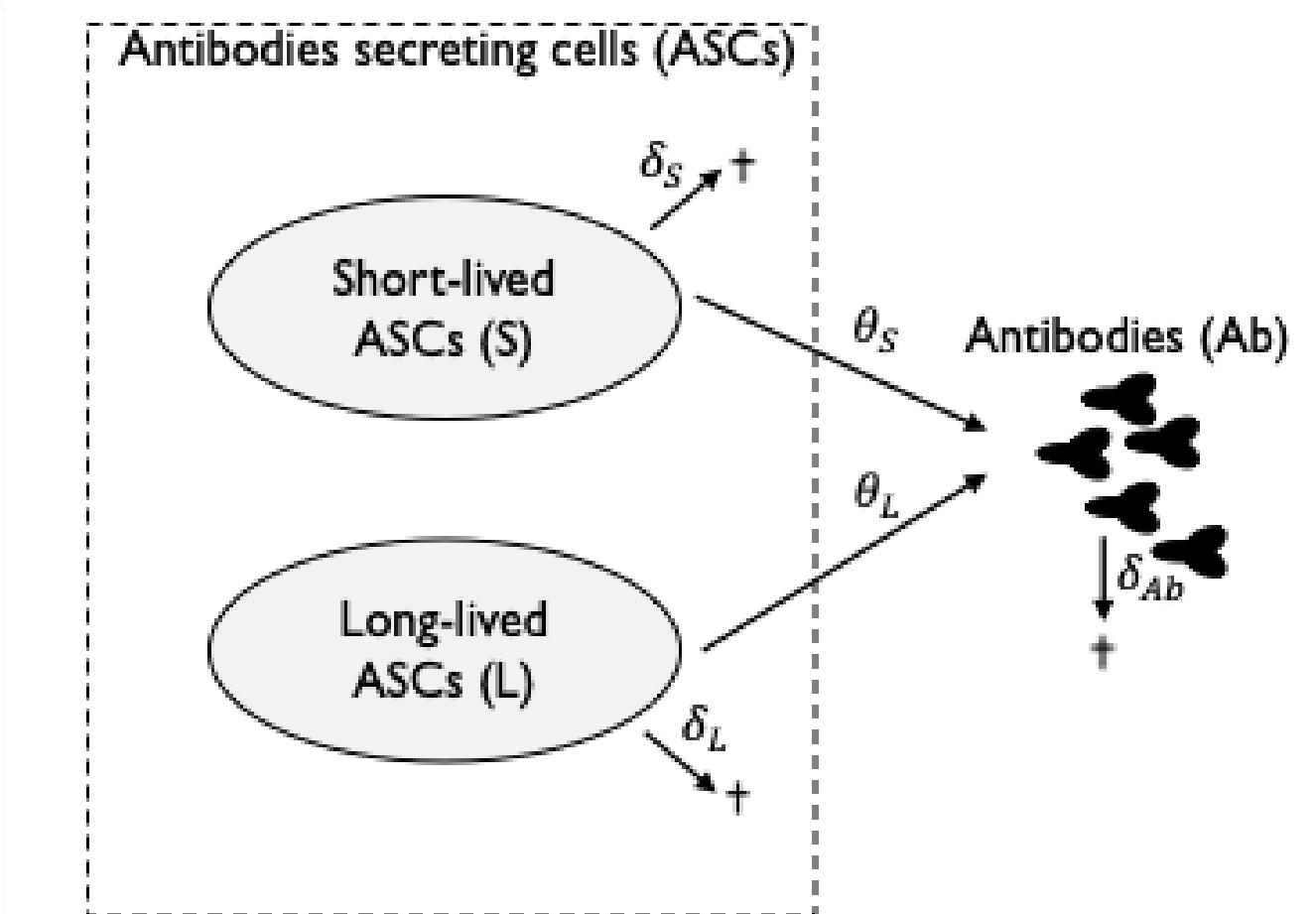
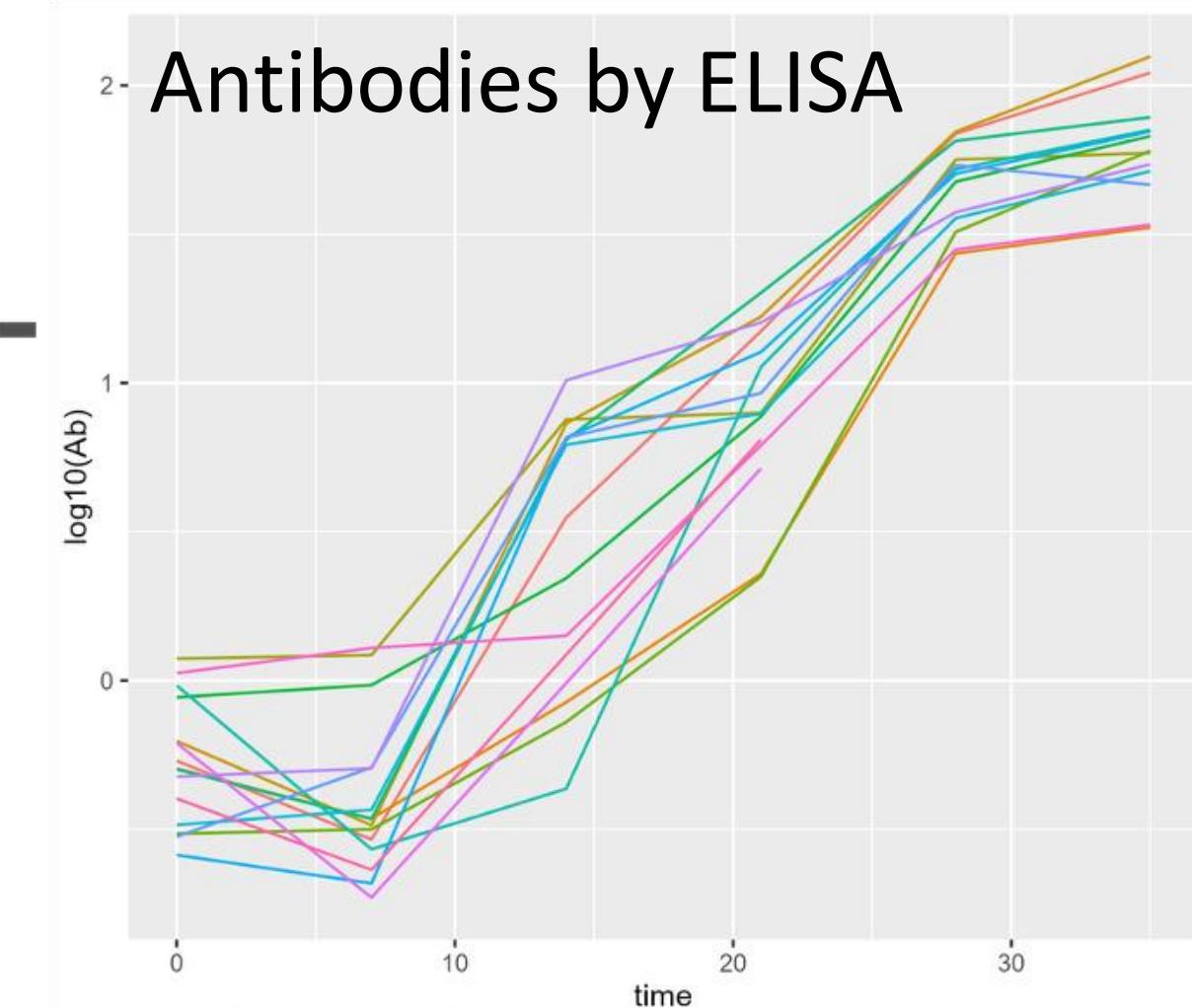
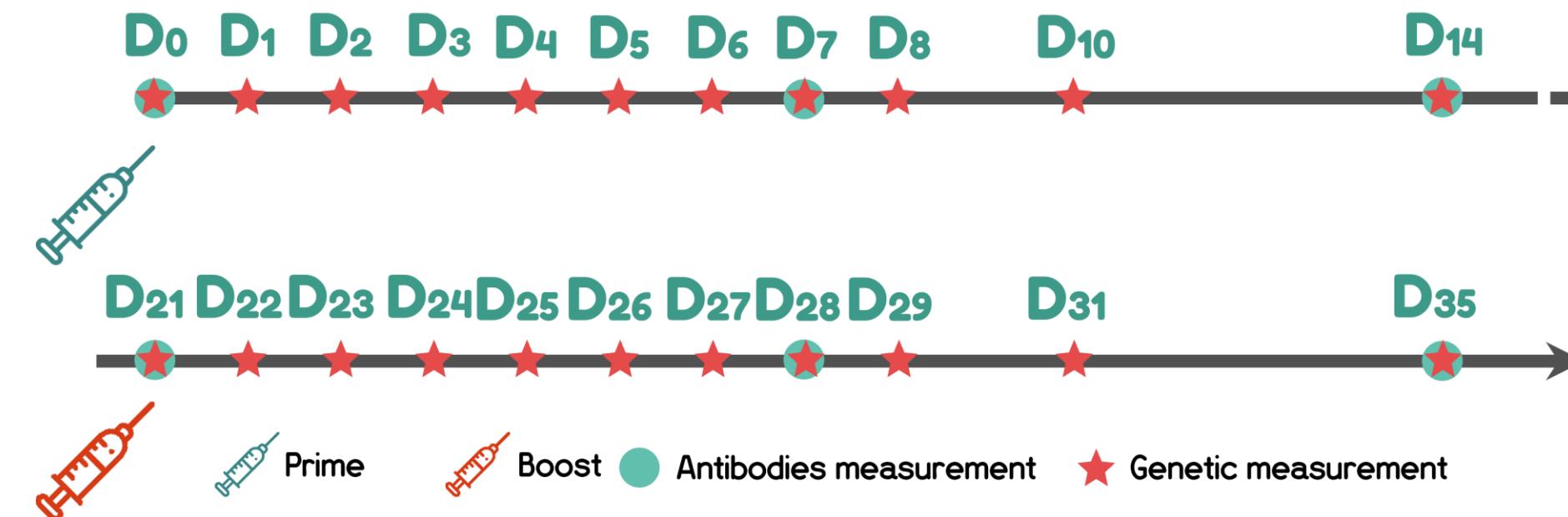
Motivating study

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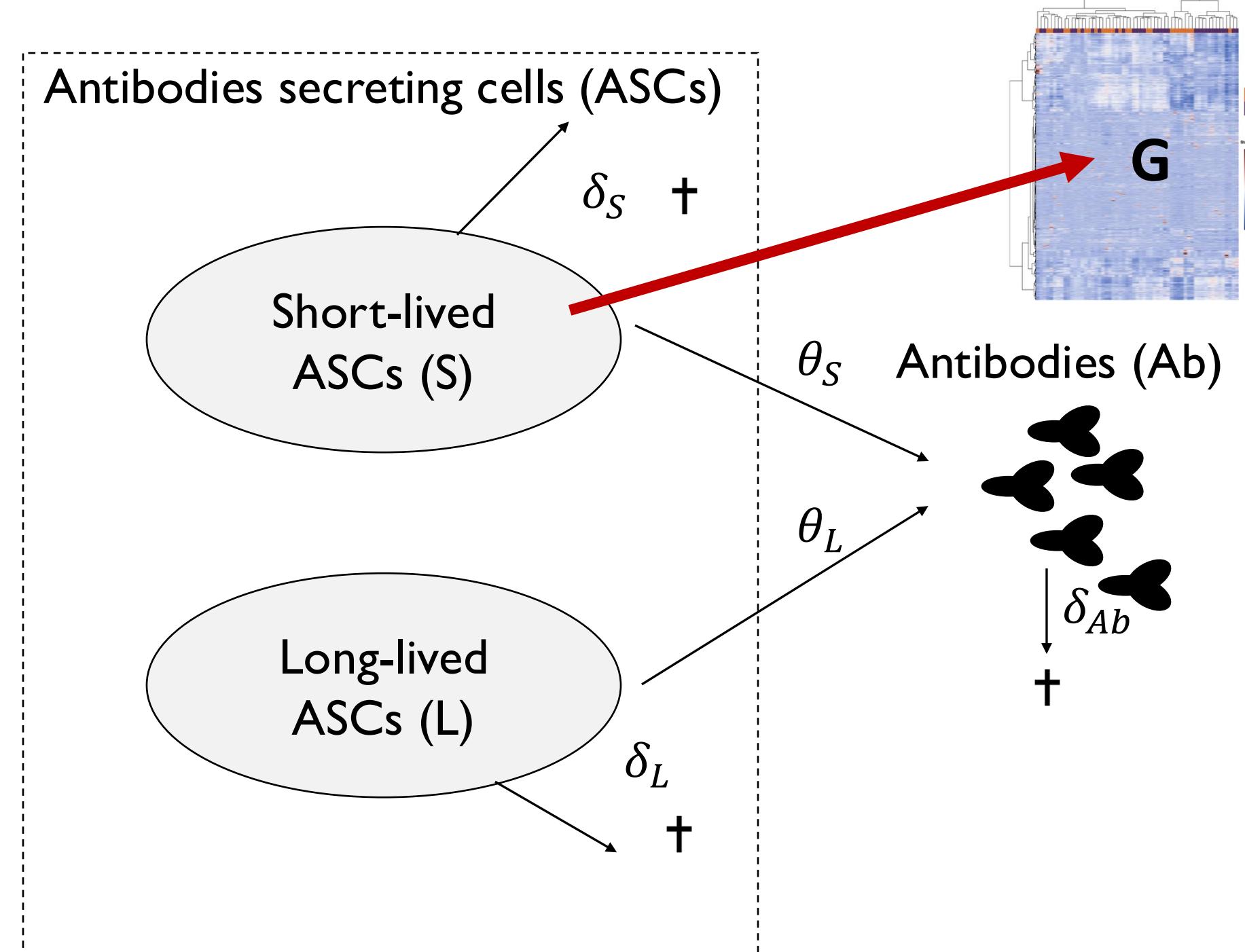


6000+ genes; 34 gene sets



Method Idea

Use longitudinal Gene expression as observation of latent compartments



Observation Model

For each individual $i \leq N$, $k \leq K$ times $(t_{ij})_{j \leq n_i}, (t_{ijk})_{j \leq n_{ki}}$:

$$Y_{ij} = h(\mathbf{Ab}_i(t_{ij})) + \epsilon_{ij}$$

$$G_{kij} = \alpha_{0k} + \alpha_{1k} S_i(t_{ijk}) + \epsilon_{ijk}$$

where $\epsilon = (\epsilon_{ij})_{i \leq N, j \leq n_i} \sim \mathcal{N}(0, \Sigma^2)$

$$\epsilon_k = (\epsilon_{ijk})_{i \leq N, j \leq n_{ik}} \sim \mathcal{N}(0, \sigma_k^2).$$

Simultaneous estimation of model parameters are $\theta = (\psi_{pop}, \beta, \Omega, (\sigma_k^2)_{k \leq K}, (\Sigma_p^2)_{p \leq P}, (\alpha_{0k})_{k \leq K})$ and the regularized parameters $\alpha = (\alpha_{1k})_{k \leq K}$ by maximizing log-likelihood under lasso penalization :

$$LL_{pen}(\theta, \alpha) = LL(\theta, \alpha) - \lambda |\alpha|$$

REMixed Algorithm – cyclic descent algorithm

$$LL_{pen}(\theta, \alpha) = LL(\theta, \alpha) - \lambda|\alpha|$$

For a given [penalty parameters \$\lambda\$](#) , the iteration l in the estimation procedure correspond to :

At iteration l :

Current parameters are $\theta^{(l)}, \alpha_1^{(l)}$.

1. Update $\alpha_1^{(l+1)}$ for fixed $\theta = \theta^{(l)}$ using [update formula derived from penalized log-ikelihood maximization](#).
2. Update $\theta^{(l+1)}$ for fixed $\alpha_1 = \alpha_1^{(l+1)}$ using [SAEM algorithm through Monolix software](#).

We continue itérations until :

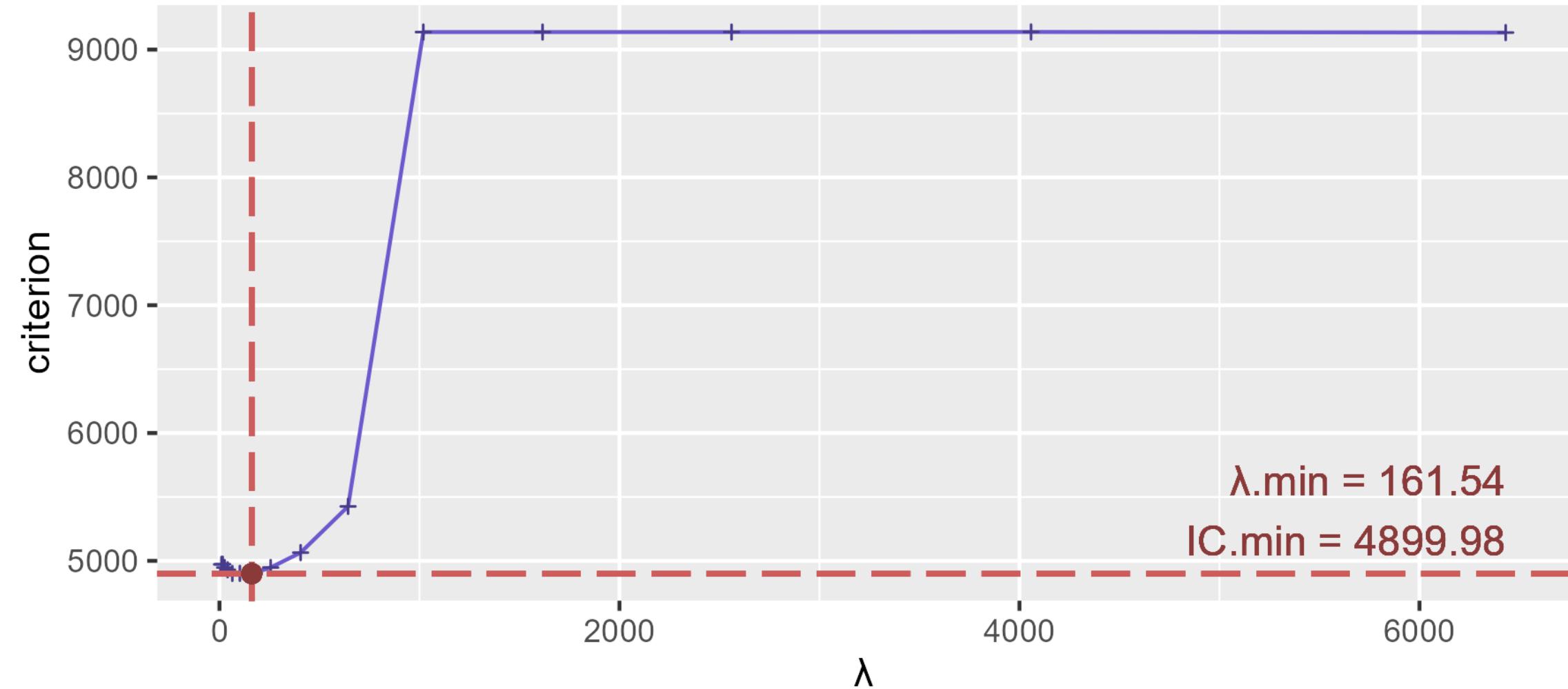
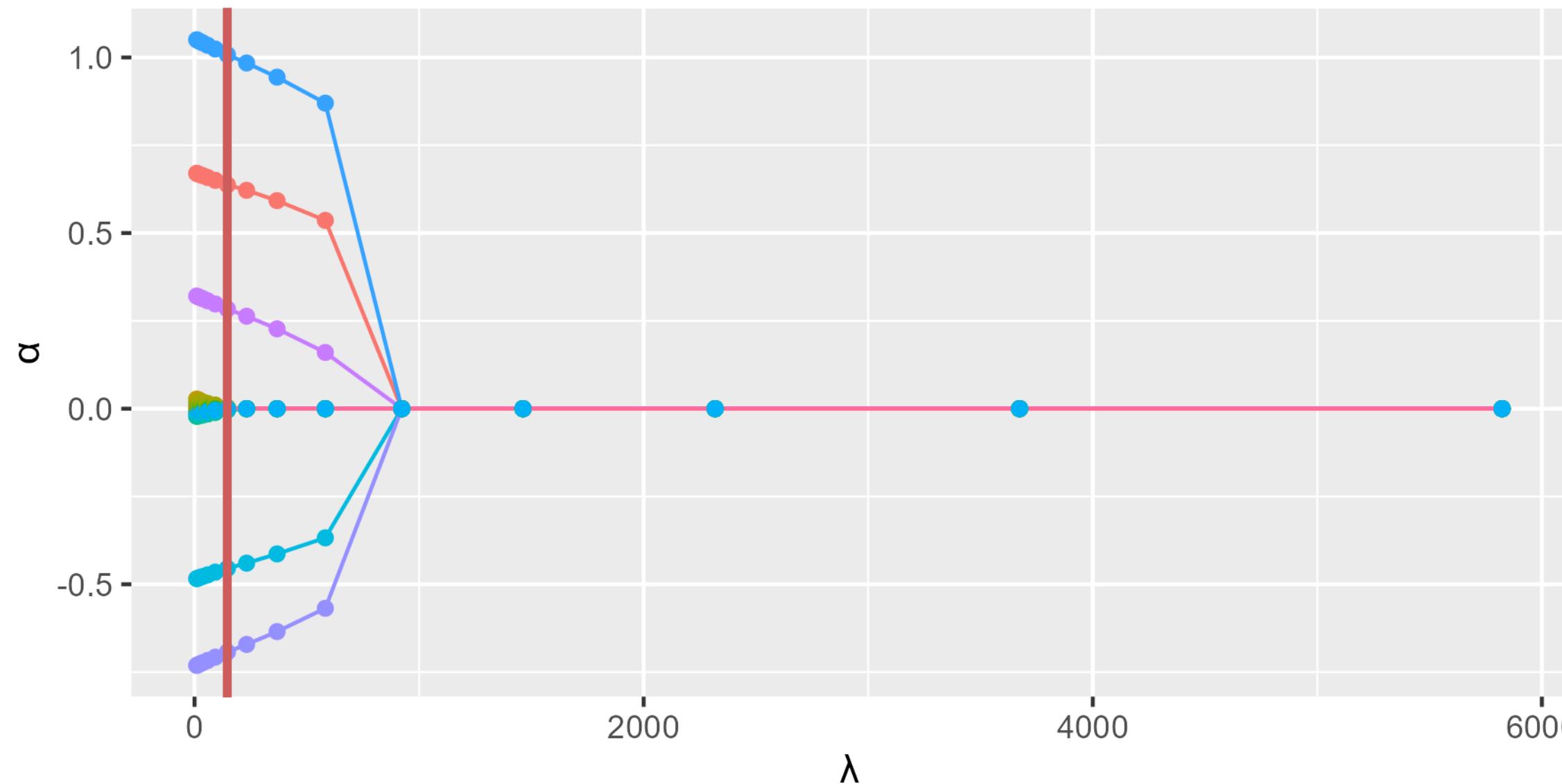
- $\left\| \left(\theta^{(l+1)}, \alpha_1^{(l+1)} \right) - \left(\theta^{(l)}, \alpha_1^{(l)} \right) \right\|_2 \leq \varepsilon_1$ (parameters stability)
- $\left| LL_{pen} \left(\theta^{(l+1)}, \alpha_1^{(l+1)} \right) - LL_{pen} \left(\theta^{(l)}, \alpha_1^{(l)} \right) \right| \leq \varepsilon_2$ (penalised log-likelihood stability)

REMixed Algorithm – Choice of lambda

Presented procedure was given a penalty parameter λ .

For $\Lambda = \left\{ \lambda_l = \lambda_{max} \times \alpha_l^{\frac{l}{N\lambda}}; 1 \leq l \leq N \right\}$, with $\lambda_{max} = \max(\partial_\alpha LL(\theta^{(0)}, \alpha) |_{\alpha=0_K})$,

$$BICc(\lambda) = -2LL(\theta^*, \alpha^*) + \log(N) \dim(\theta_R) + \log(n_{tot}) \dim(\theta_F)$$



A final SAEM is then computed followed by statistical test to remove non-significant biomarkers.

Simulations setting

Structural Model 25 individuals, 50 biomarkers – 10 informative biomarkers , 200 replicates

$$\dot{S}_i(t) = -\delta_{S_i} S_i(t)$$

$$\dot{Ab}_i(t) = \varphi_{S_i} S_i(t) - \delta_{Ab_i} Ab_i(t)$$

$$S(t = 0) = 5$$

$$Ab(t = 0) = 1000$$

$$\log(\delta_{S_i}) = \log(\delta_{S_{pop}}) + \eta_i^S$$

$$\log(\delta_{Ab_i}) = \log(\delta_{Ab_{pop}}) + \eta_i^{Ab}$$

$$\log(\varphi_{S_i}) = \log(\varphi_{S_{pop}}) + \eta_i^\varphi$$

Statistical Model

$$\eta_i^\chi \sim \mathcal{N}(0, \omega_\chi^2), \quad \chi \in \{S, Ab, \varphi\}$$

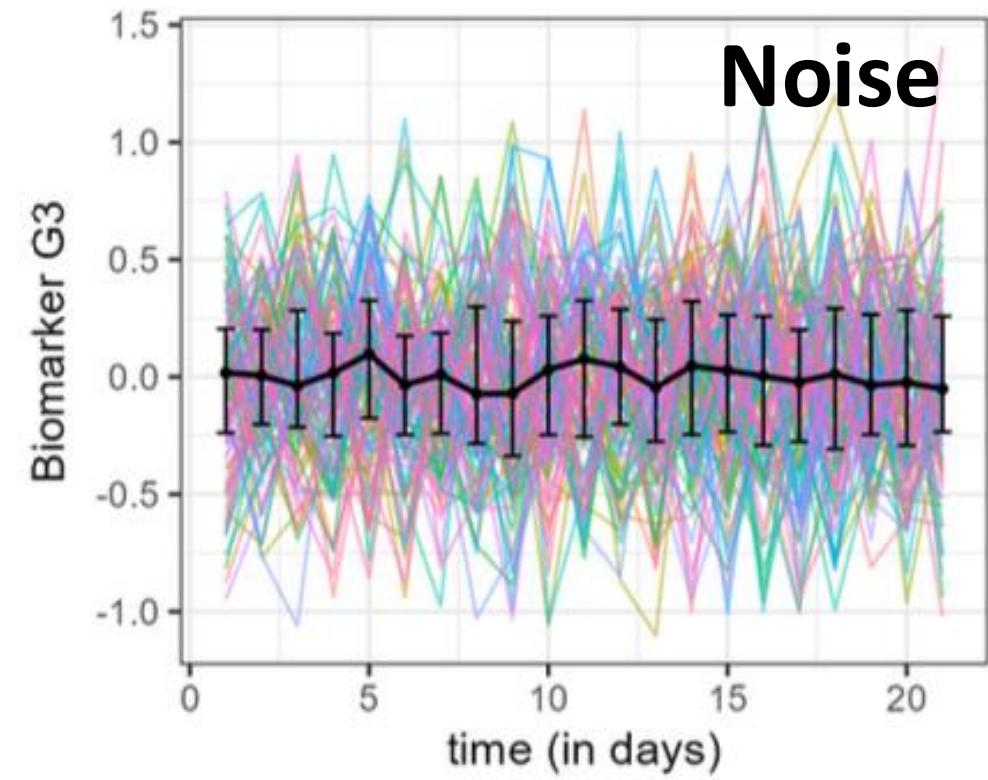
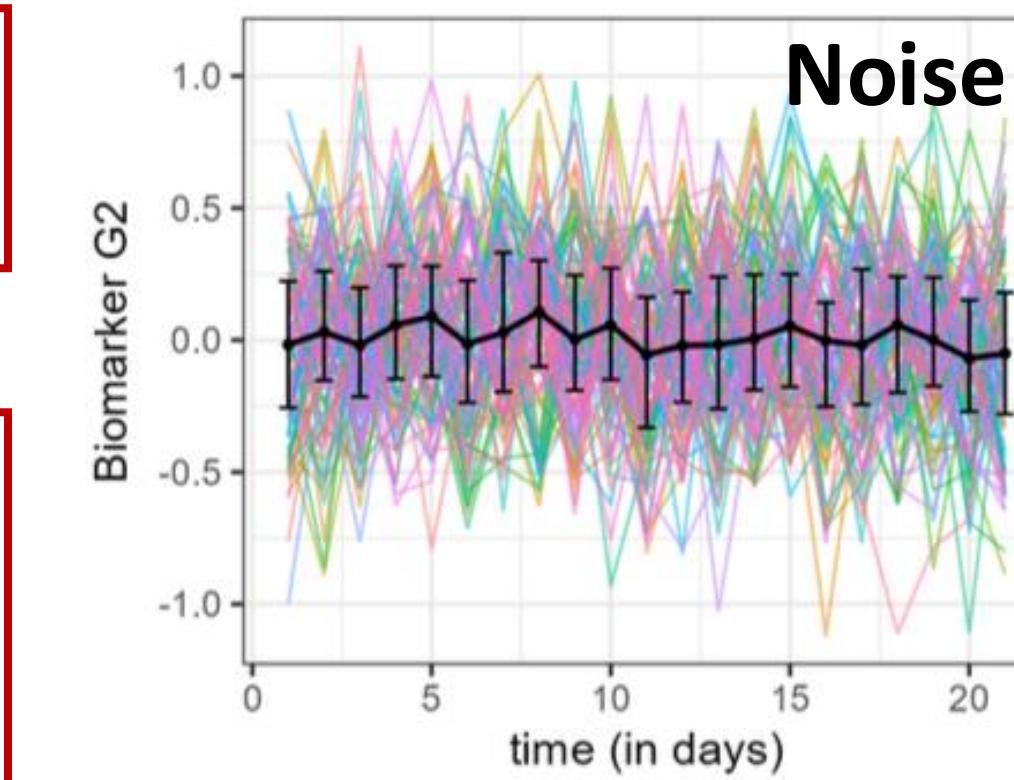
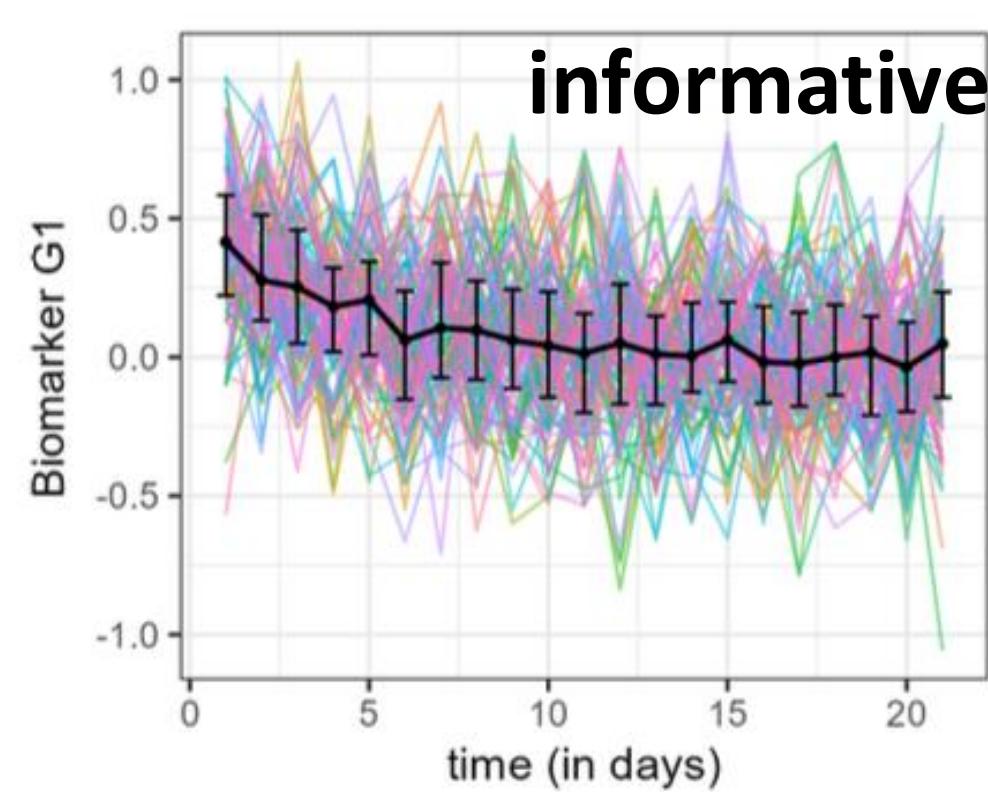
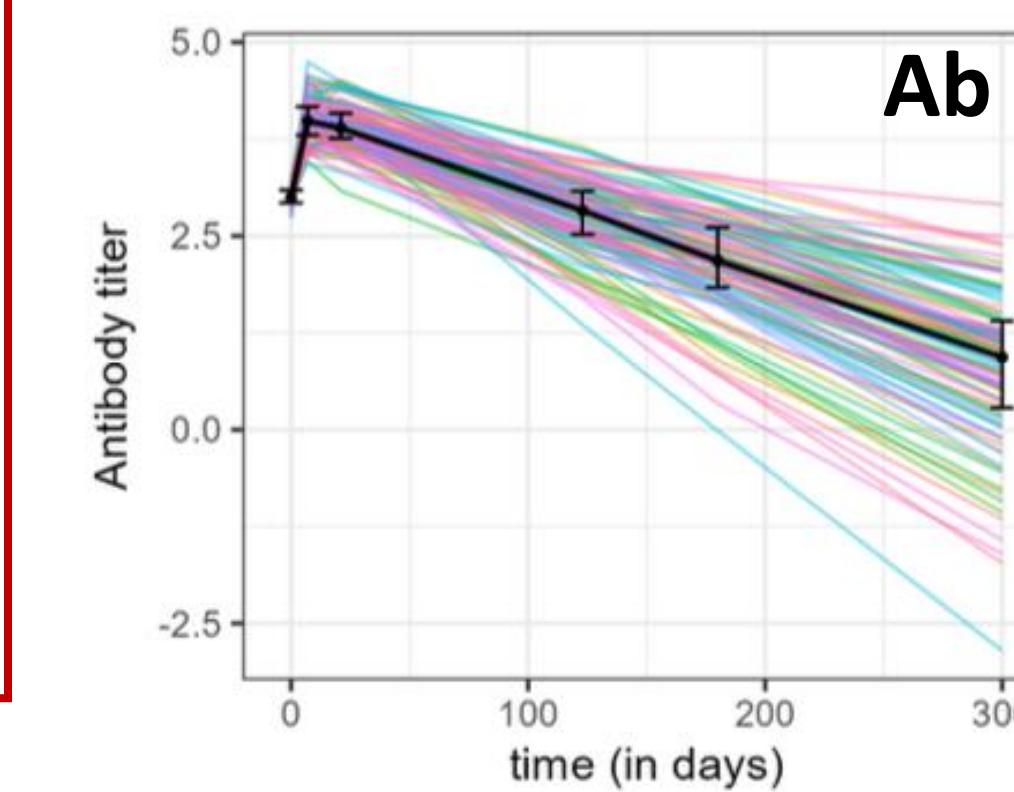
Observation Model $t_j \in \{0, 7, 21, 123, 180, 300\}; t'_j \in \{0, \dots, 21\}$

$$Y_{ij} = \log_{10}(Ab_i(t_j)) + \varepsilon_{ij}$$

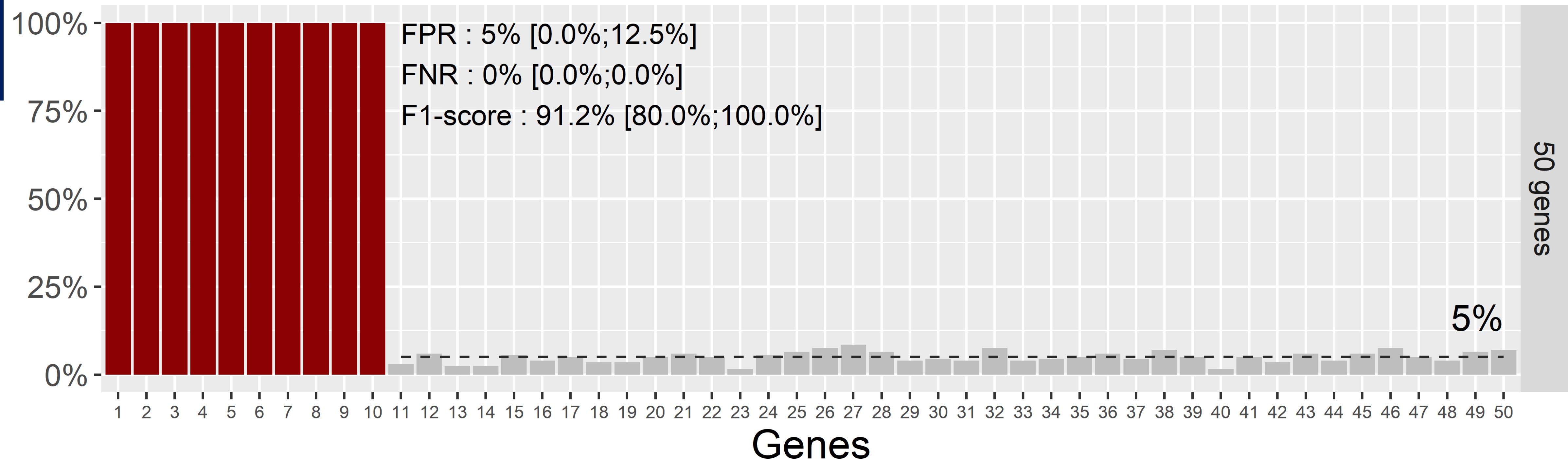
$$G_{kij} = \alpha_{0k} + \alpha_{1k} S_i(t'_j) + \varepsilon_{kij}$$

$$(\varepsilon_{ij}) \sim \mathcal{N}(0, \sigma_{Ab}^2) \text{ iid}$$

$$(\varepsilon_{kij}) \sim \mathcal{N}(0, \sigma_{G_k}^2) \text{ iid}$$



Simulations results

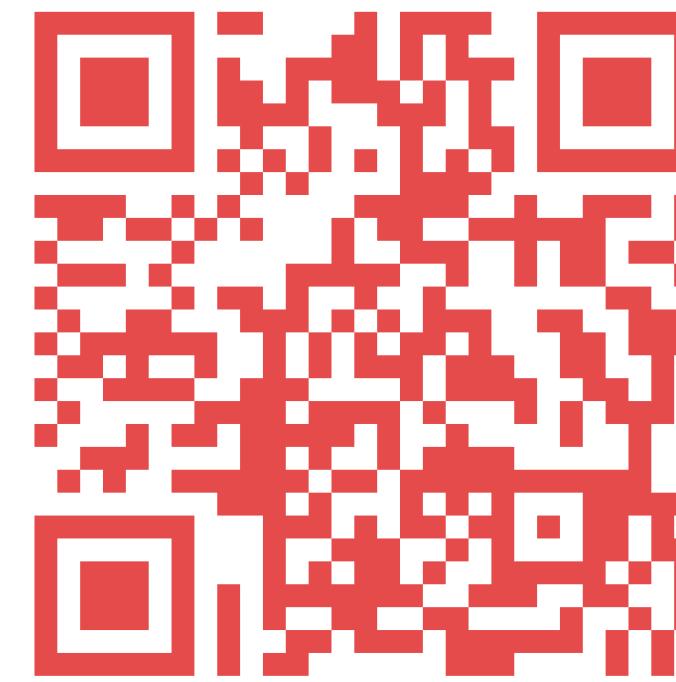


04

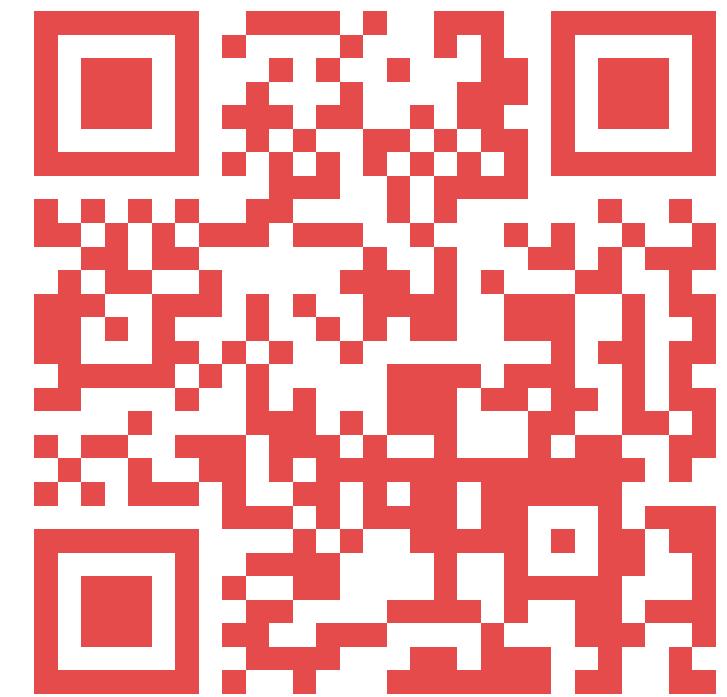
Conclusions and perspectives

Perspectives

- ❑ Packages lasso-SAMBA and REMixed have been released on CRAN



CRAN lasso-SAMBA



CRAN REMixed

- ❑ Find relevant application of these methods (including in pharmacogenomics) and evaluate if assumptions can/have to be relaxed (ie. Linear relationship).

$$\text{Lasso-SAMBA } g(\theta_i(t)) = g(\theta_0) + \mathbf{h}(X_i(t)) + u_i$$

$$\text{REMixed } G_{kij} = \mathbf{h}(S_i(t'_j), \varepsilon_{kij})$$

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 **inria** inventeurs du monde numérique

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 **ISPED** Bordeaux School of public health
Institut de Santé Publique d'Epidémiologie et de Développement **Université de BORDEAUX**

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Simulation+ / Lixoft Monolix Suite

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Thank you !

WE ARE HIRING POSTDOCS IN BIOSTATISTICS
/ MODELING / PHARMACOMETRICS
(MELANIE.PRAQUE@INRIA.FR)



2

Method : Lasso Selection

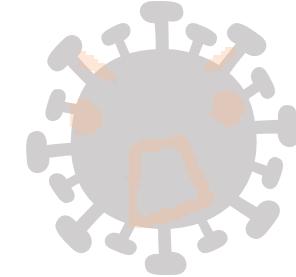
Individual Parameter regression :

We have previously build model \mathcal{M}_k at iteration k , with parameters $\theta^{(k)}$, $\left(\psi_i^{(k)}\right)_{i \leq N}$; we write the regression model **for each parameter** :

$$g\left(\psi_i^{(k)}\right) = g_l\left(\psi_{pop}^{(k)}\right) + \beta X_i$$

2

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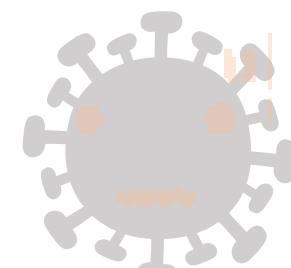
$$g\left(\psi_i^{(k)}\right) = g_l\left(\psi_{pop}^{(k)}\right) + \beta X_i$$

Lasso Regression :

We then compute the lasso estimator

$$\hat{\beta} = \arg \min_{\beta \in \mathbb{R}^n} \left\{ \sum_{i \leq N} \left(g\left(\psi_i^{(k)}\right) - g\left(\psi_{pop}^{(k)}\right) - \beta X_i \right)^2 + \lambda |\beta|_1 \right\}$$

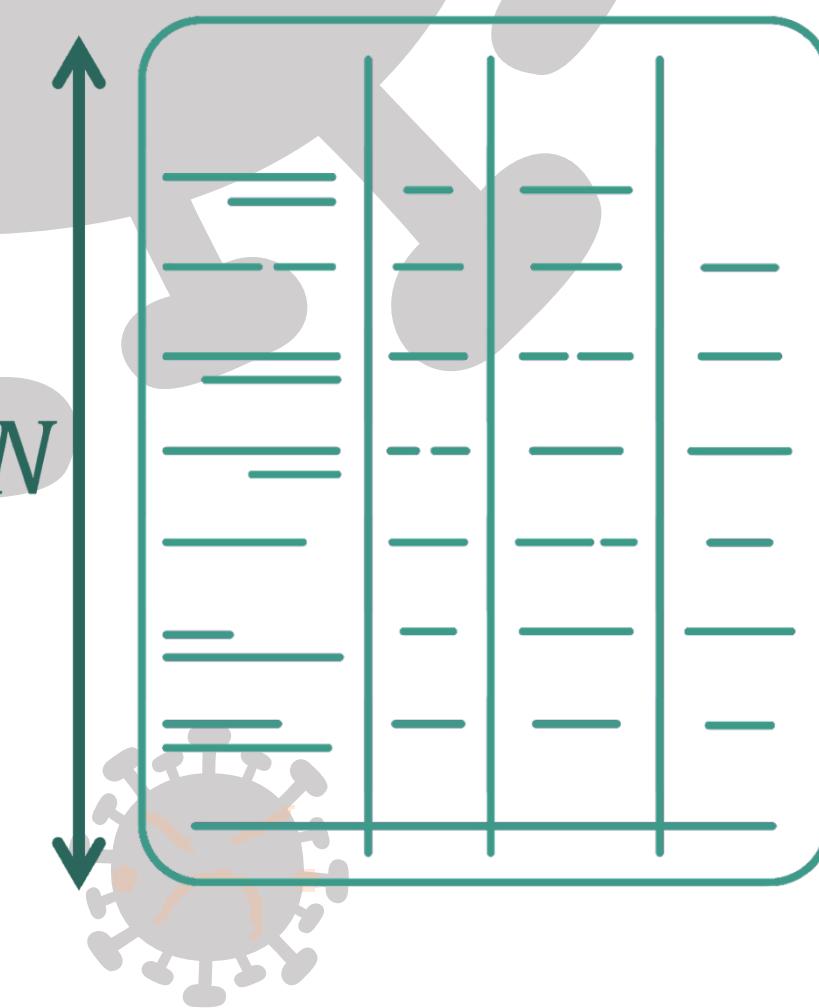
Where $\lambda > 0$ is a data-driven penalization parameter.



2

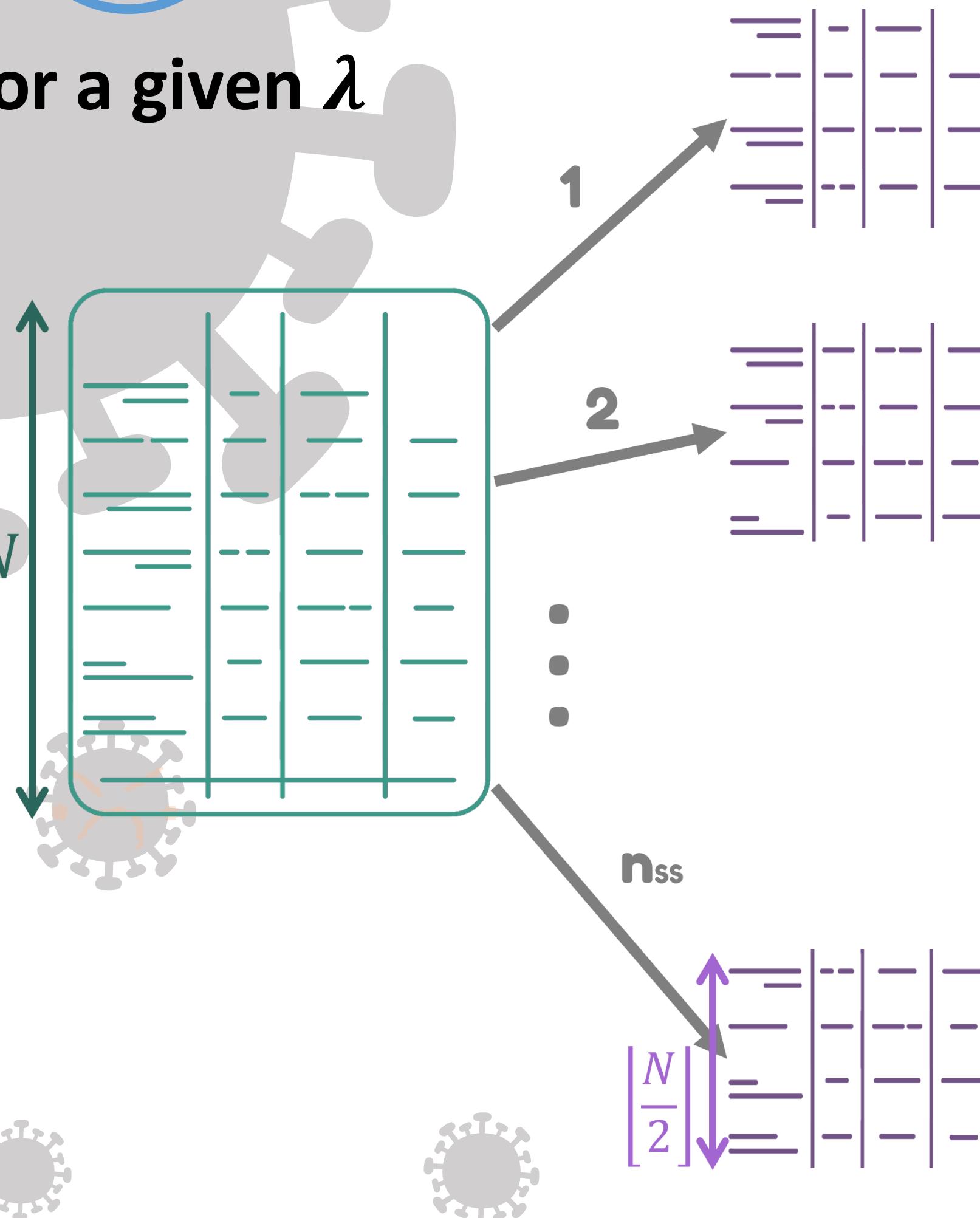
Method: Stability Selection

For a given λ



② Method

For a given λ

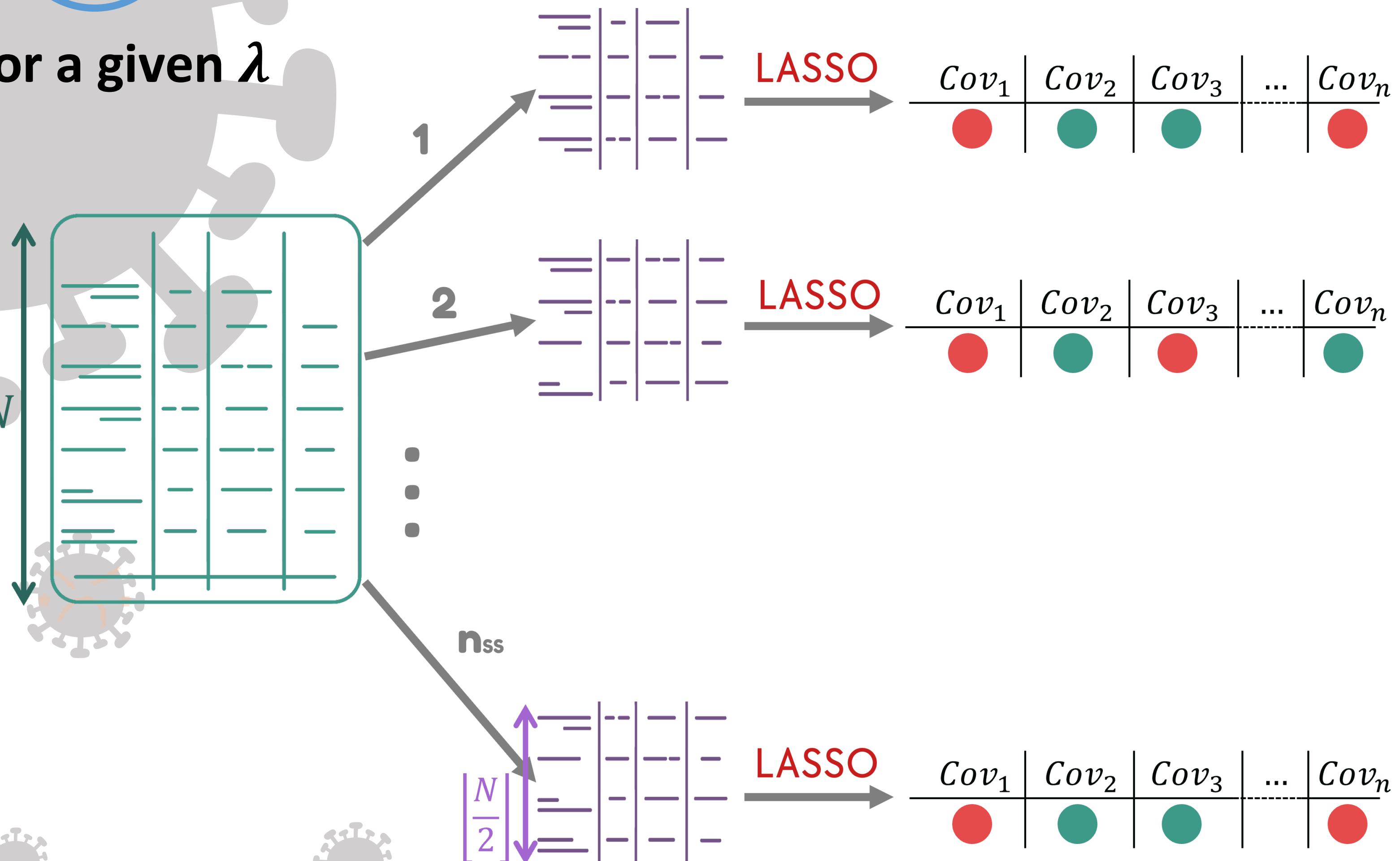


Method: Stability Selection

2

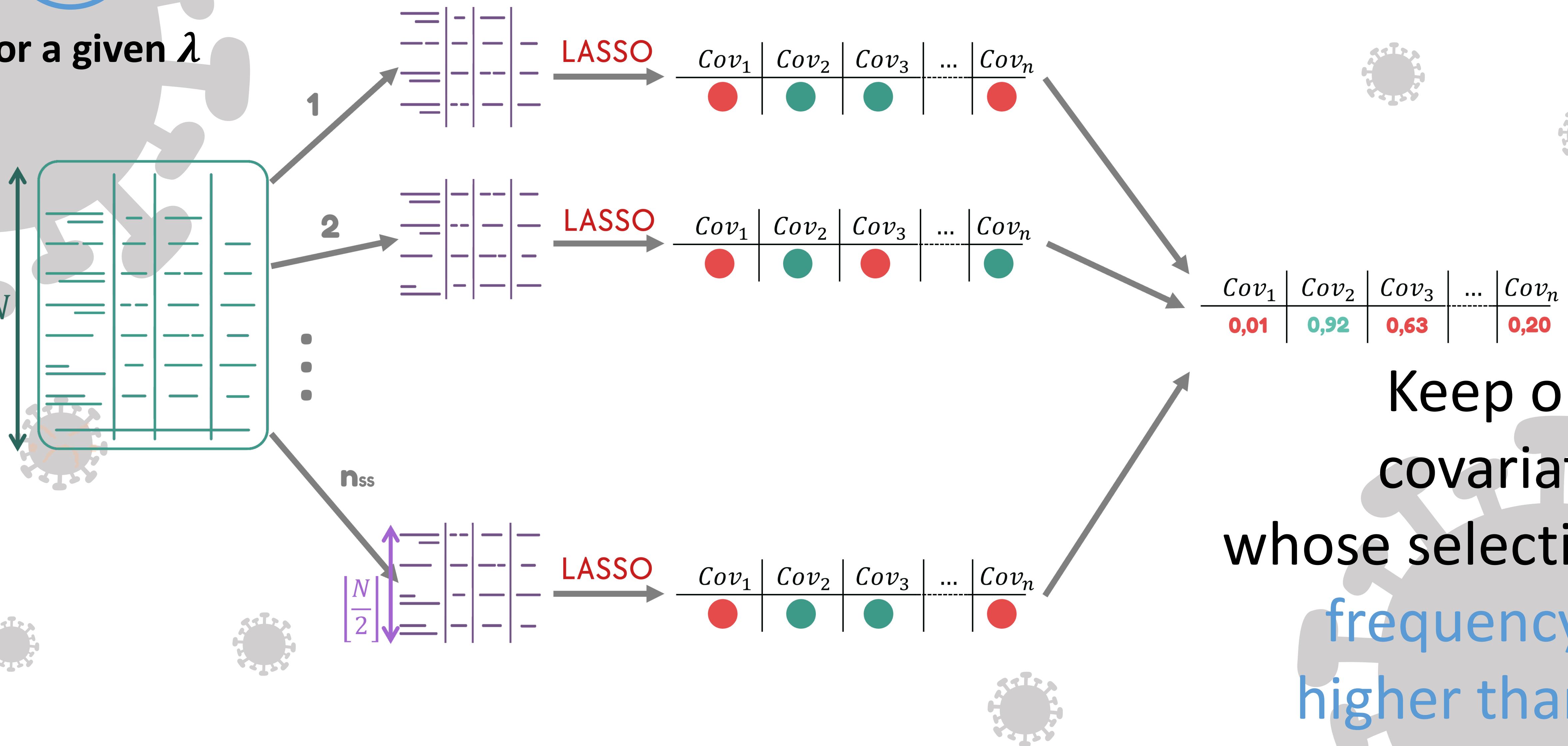
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2

Method: Parameters calibration and IC

Lasso selection enhanced by stability selection depends on two parameters

λ the penalization parameter
 t_{ss} the selection thresholds

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- ! Decreasing IC in covariate selection step by stepAIC in SAMBA seems to allows algorithm to stop.
 - > Rather than one single model, we construct a set of relevant model to explore, searching for one decreasing the IC.

2

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> Rather than one single model, we construct a set of relevant model to explore, searching for one decreasing the IC.

Meinshausen and Bühlmann, 2010 :

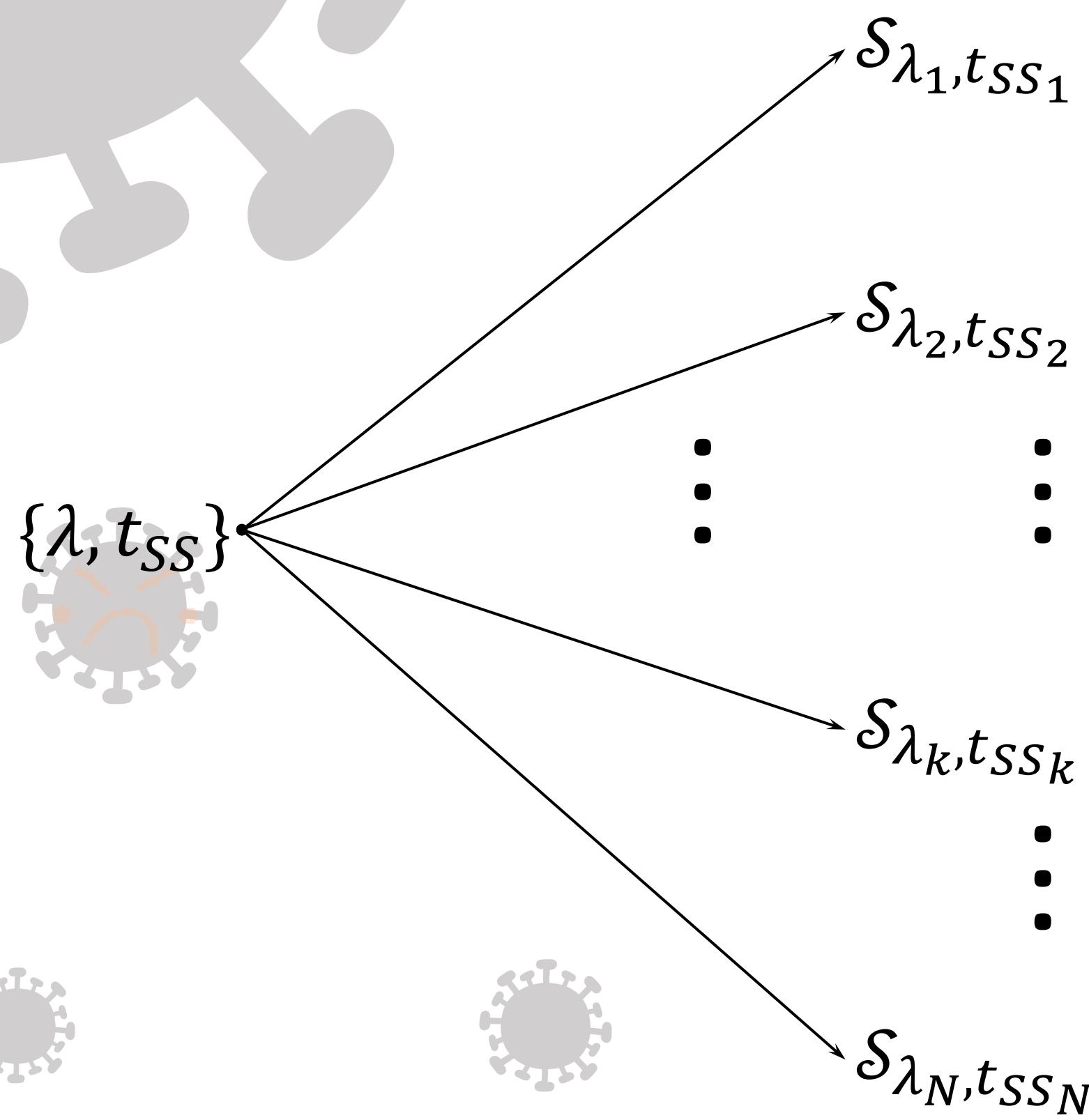
$$\mathbb{E}[\#FP(\mathcal{S}^{\lambda, t_{SS}})] \leq U_{\lambda, t_{SS}} = \frac{1}{2t_{SS} - 2} \times \frac{q_{\lambda}^2}{n}$$

Where q_{λ} is the average number of features that are selected at least once by the Lasso algorithm, and n the number of covariates.

2

Method: Parameters calibration and IC

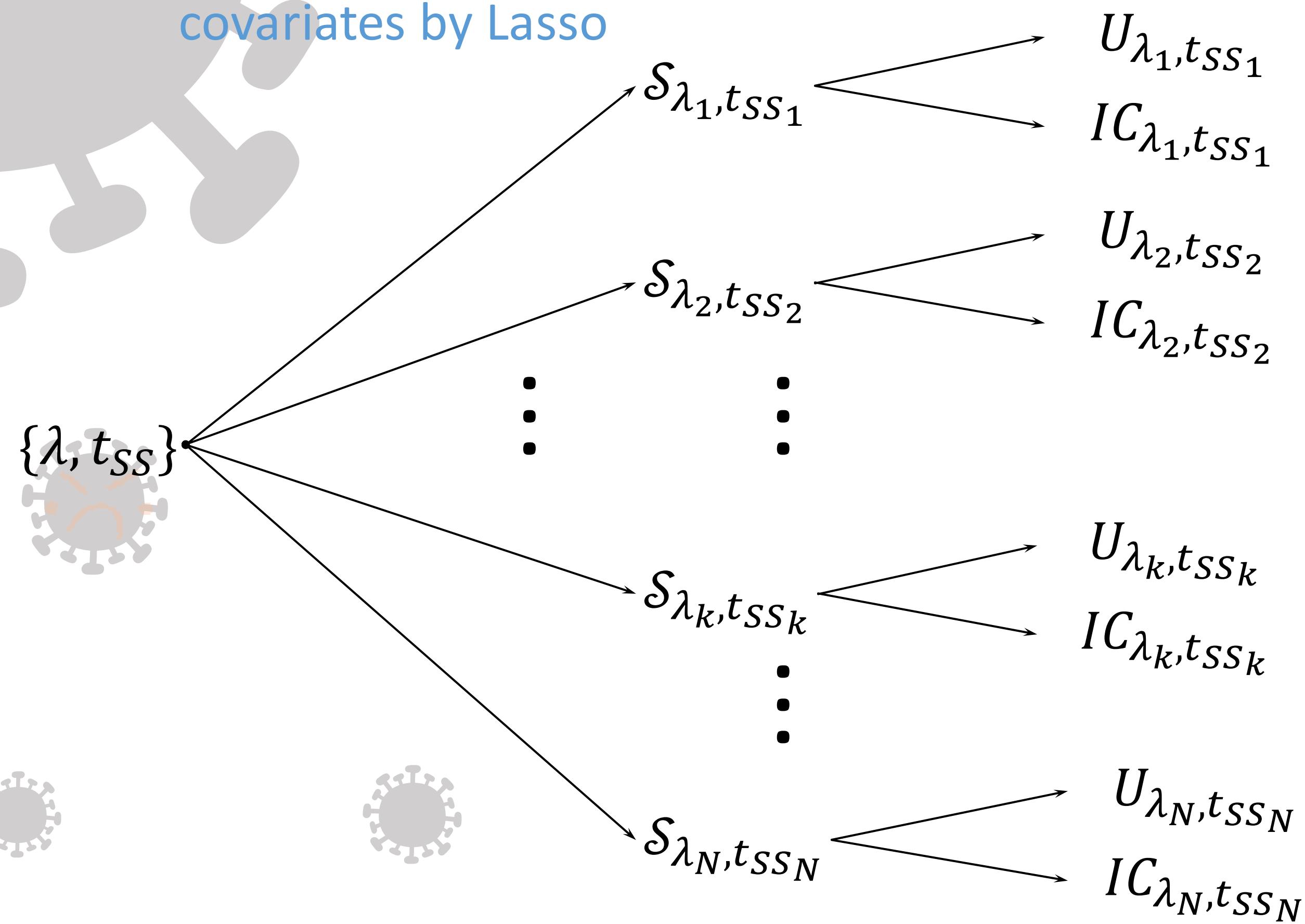
Selected set $\mathcal{S}_{\lambda, t_{SS}}$ of covariates by Lasso



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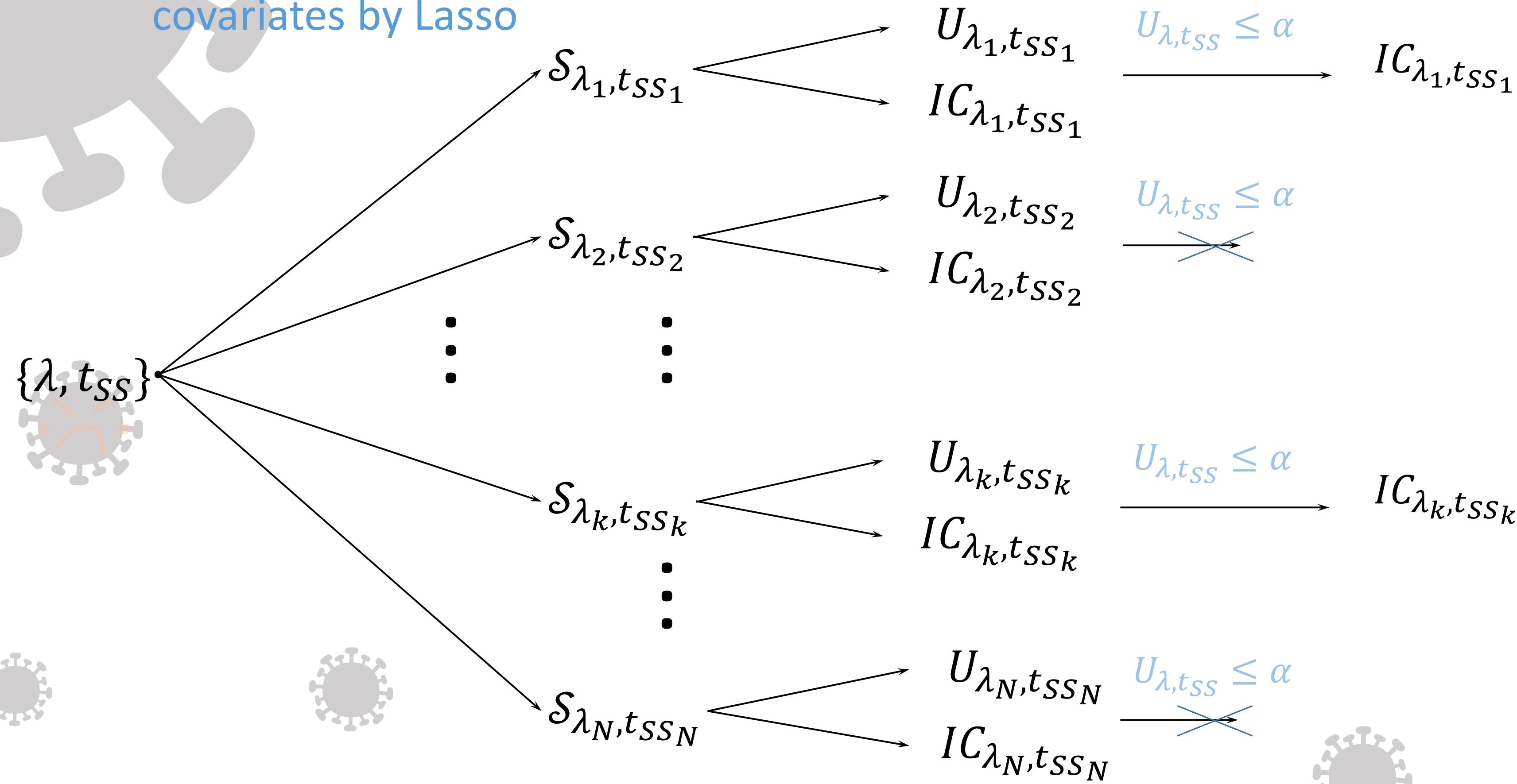
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Selected set $\mathcal{S}_{\lambda, t_{SS}}$ of covariates by Lasso



2

Method : REMix algorithm

Update formula of regularized parameters.

At this step, we suppose fixed $\theta^{(l)}$, and current estimate $\alpha^{(l)}$

We want to solve

$$\partial_{\alpha_1} (LL(\theta^{(l)}, \alpha) - \lambda|\alpha|) = 0$$

We use a second order Taylor development

$$LL(\theta^{(l)}, \alpha) = LL(\theta^{(l)}, \alpha^{(l)}) + (\alpha - \alpha^{(l)})^T \partial_{\alpha} LL(\theta^{(l)}, \alpha) \Big|_{\alpha=\alpha^{(l)}} - \frac{1}{2} (\alpha - \alpha^{(l)})^T \partial_{\alpha}^2 LL(\theta^{(l)}, \alpha) \Big|_{\alpha=\alpha^{(l)}} (\alpha - \alpha^{(l)})$$

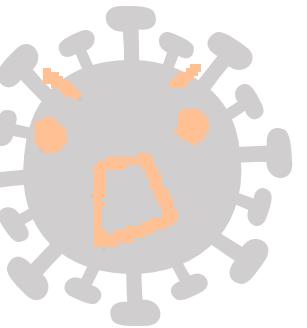
The update formula for regularized parameters $\alpha_{1k}^{(l+1)}$:

$$\alpha_{1k}^{(l+1)} = \begin{cases} \frac{A + \lambda}{\partial_{\alpha_{1k}}^2 LL(\theta^{(l)}, \alpha) \Big|_{\alpha=\alpha^{(l)}}}, & \text{if } A < -\lambda \\ \frac{A - \lambda}{\partial_{\alpha_{1k}}^2 LL(\theta^{(l)}, \alpha) \Big|_{\alpha=\alpha^{(l)}}}, & \text{if } A < +\lambda \\ 0, & \text{otherwise.} \end{cases}$$

With A a function of gradient, hessian and current estimates.

3

Simulation Studies



Initialization Strategy

- We randomly group genes by pairs and run a SAEM algorithm for each pair.
- The initialization yielding the best log-likelihood is selected to launch the final algorithm.

